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WALKING WHEEL

V.V. Lapshin lapshin032@bmstu.ru

Bauman Moscow State Technical University, Moscow, Russian Federation

Abstract

A hypothesis was proposed that during the bipedal walking, there appear stable periodic movements in certain variables (self-oscillations). In this case, it is possible to easily change parameters of this periodic locomotion using open (without feedback) control loops with respect to some of the variables. As the first stage in testing this hypothesis, dynamics of the walking wheel downward movement along an inclined plane was analytically studied. Walking wheel is the simplest model of passive bipedal walking. When it moves, energy is supplied to the system due to the force of gravity action. It is shown that point mapping of the wheel angular speed alteration per step (Poincaré map) in the overwhelming majority of cases has one fixed point. This fixed point corresponds either to stable periodic solution (self-oscillation), which is the wheel rolling down an inclined plane, or to the wheel movement ending with its termination as a result of the endless series of impacts with swinging on two legs. In the degenerate case, the Poincaré map has two fixed points. One of them corresponds to the unstable limiting cycle matching the wheel rolling, and the second corresponds to a wheel stop. In this case, the limiting cycle is stable outside and unstable inside itself

Keywords

Nonlinear dynamics, bipedal walking, walking wheel, rimless wheel

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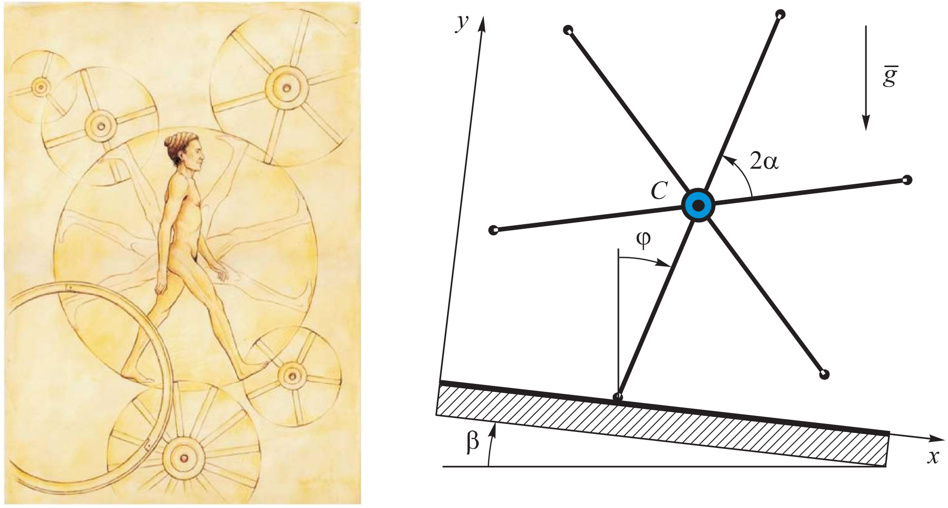
Introduction. The problem of creating and controlling the motion of automatic bipedal devices capable of moving like a person attracted attention of scientists and engineers since the time of Leonardo da Vinci. Various approaches are known to solving the task of controlling the motion of bipedal devices. Let us consider some of them. This is construction of periodic movements in terms of variables [1], use of “rigid” tracking systems [2], pulse control [3–5], methods of controlling systems with zero dynamics [6] and passive bipedal walking [7–8]. Results presented in [9–10] are important milestones in understanding the issues of motion control and creation of real layouts of bipedal robots.

If a bipedal robot has big controllable feet, its movement could be organized within the frames of static stability. However, to organize the motion of a bipedal robot capable of moving within the frames of dynamic stability, including a device without controllable feet, control system is forced to solve a very complex problem of controlling a mechanical system with a lack of controls. In this case, it is impossible to ensure arbitrary programmed motion along all degrees of freedom of the robot. Periodic programmed motion is usually constructed, and algorithms for stabilizing this motion are elaborated. When a robot capable of changing parameters of its motion is moving, the motion control system has to integrate differential equations of the its motion even when moving over a simple terrain (flat horizontal surface or surface with slight irregularities). At the same time, a moving person does not integrate differential equations of motion, and, when moving on a simple and relatively flat terrain, a person does not think at all about the process of organizing such walking. Everything is carried out at the subconscious level. It could be assumed that control over the bipedal walking may be carried out much easier. Perhaps, walking is a constant process of falling, when a person substitutes another leg to prevent a fall. However, the motion parameters could be easily changed. The motion control system is built quite simply due to existence of stable periodic motions in terms of variables (self-oscillations). To control motion in terms of certain variables, open (without feedback) control loops could be used in the same way as it is done in the passive dynamic walkers [7–8] and hopping robots with elastic elements in the leg structure [11–13]. Parameters of these stable periodic solutions (self-oscillations) could alter due to the operating independent motion control loops along other degrees of freedom of the robot.

This work is the first step trying to clear up this hypothesis. Let us start with studying an object that is much simpler than a bipedal walking device, i.e., a walking wheel, or a wheel with legs (Fig. 1), proposed by A.M. Formalsky [3]. This model was considered independently of him somewhat later by T. McGeer [7]; the obtained result was used to create a device demonstrating passive bipedal walking without the use of drives in the leg joints. Walking wheel is the simplest model of the flat bipedal walking device motion. If during motion the bipedal device (person) substitutes a new leg not to fall forward, then the walking wheel next leg gets in contact with the supporting surface as a result of its rotation (rolling). This model of the walking device motion is of interest due to its simplicity. Plane motion of a rigid body is under study, while a walking device (person) appears to be a system of several rigid bodies with drives in hinges connecting these bodies. Walking wheel was also proposed to be used by D.J. Todd [14] in the design of wheeled off-road robots. Walking wheel



a *b*

Fig. 1. Walking wheel as a model of bipedal walking* (*a*) and design diagram of a walking wheel on the inclined plane (*b*)

resembles ship steering wheel, gear, cart rimless wheel, rigid wheel with large ground lugs, or a wheel with very rough tread when moving on a hard surface. Foreign literature uses the term *rimless wheel.*

When the foot of a walking wheel is placed on the supporting surface, an impact occurs, and it is assumed that this is an absolutely inelastic impact, and the leg is not sliding on the supporting surface. When the impact occurs, loss of energy is taking place; as a result, when moving on a horizontal surface, motion speed slows down, and the wheel stops [3]. When moving down along the inclined plane with sufficiently wide inclination angle to the horizon, the wheel gains stable periodic motion mode (self-oscillation). Energy consumption by the system is ensured by the work of gravity force. This process was approximately investigated in [7] in a linearized model of motion under assumption that the angle between legs of the walking wheel and the angle of surface inclination to the horizon are small. This problem is analytically solved here in the nonlinear setting.

Results of studying the walking wheel locomotion by methods of simulating its movement on a computer and prototyping are presented in [15–19]. Interactive mathematical model of the walking wheel motion with animation of its movement, construction of phase trajectories and possibility to set parameters of the wheel and of the slope inclination angle, along which it moves, is available on the Internet [19].

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* Source: author unknown.

Problem statement. Let us consider a solid body (Fig. 1 *b*), i.e., a disk, to the edge of which the n identical rods are attached, $n \geq 3$. Outer ends of the rods form a regular polygon. Let us call this body a walking wheel. Segment connecting the C center of mass of the body with the outer end of the rod would be called a leg (virtual leg). The outer end of the rod would be called a foot. Suppose that the lengths of all legs are the same and are equal to l. The angle between adjacent legs is 2α, where $\alpha = \pi / n$. Mass is equal to m, $J_C = m\rho^2$ is the moment of inertia relative to the center of mass, ρ is the radius of gyration relative to the center of mass, $\rho > 0$. Wheel position is determined by the rotation angle φ, which is measured from the vertical to the supporting leg. Clockwise direction is taken as the positive direction for angle measurement.

Let us consider flat motion of the walking wheel down an inclined plane with the β inclination angle to the horizon. Suppose that the supporting surface is absolutely rough, and the feet are unable to slide on it. Speed of the center of mass is denoted as $\bar{V}$, and angular speed of the wheel — as ω. Motion of the wheel consists in alternation of rotation phases around the supporting leg foot and of impacts, when the supporting legs are changed.

Impact accompanying the change in supporting legs. Before the impact (Fig. 2), the wheel rests on the support surface at point S_{i-1}, which possesses zero speed. As a result of the solid body rotation around this point, an impact occurs against the supporting surface at the new point of contact S_i. In this case, the impact is assumed to be absolutely inelastic, the new point of contact after the impact remains on the supporting surface.

Due to the fact that the φ wheel rotation angle is measured from the vertical to the supporting leg, its value changes upon impact (placing a new leg on the supporting surface) (see Fig. 2):

$$\varphi^- = \beta + \alpha, \ \ \varphi^+ = \beta - \alpha.$$

Here and after, the "+" subscript denotes value after the impact, and the "–" subscript denotes value before the impact.

Impact pulses of the supporting surface reaction arise both at the collision point S_i, as well as at the support point S_{i-1}. As a result of the impact, the initial support point could remain on the supporting surface, or may leave it, due to the fact that these constrain is unilateral.

Let us introduce the following notations $\bar{V}_i^- = (\dot{x}_{Ci}^-, \ \dot{y}_{Ci}^-)$, $\bar{V}_i^+ = (\dot{x}_{Ci}^+, \ \dot{y}_{Ci}^+)$, ω_i^-, ω_i^+ is the speed of the C center of mass and the angular wheel

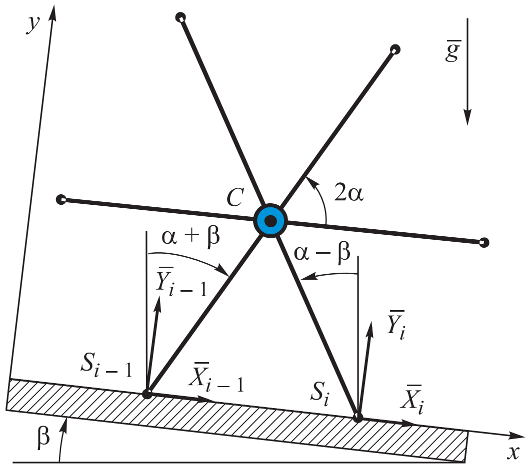

Fig. 2. Impact diagram of the walking wheel second leg contacting with the supporting surface

speed before and after the impact. Clockwise direction is taken as the angular speed positive direction and is the positive direction of the φ, wheel rotation angle measurement.

Shock pulses appear at the impact moment at the leg supporting points (shock reactions of the supporting surface); let us denote their projections on the coordinate axes as X_{i-1}, Y_{i-1} in the S_{i-1} foot and X_i, Y_i in the S_i foot (see Fig. 2).

There are unilateral constrains on coordinates of the supporting leg feet are uncontrolled (supporting leg feet could lose contact with the supporting surface and move upward). Consequently, vertical components of shock reactions at the leg support points are not negative:

$$Y_{i-1} \geq 0, \;\; Y_i \geq 0. \tag{1}$$

If the S_{i-1} foot loses contact with the supporting surface as a result of the impact, then after the impact the foot speed is directed upward:

$$\dot{y}^{+}_{S_{i-1}} = \omega_i^{+} \cdot 2l \sin\alpha > 0. \tag{2}$$

In addition, let us accept the hypothesis that shock reactions at the support point of this leg are equal to zero [3]:

$$X_{i-1} = 0, \; Y_{i-1} = 0. \tag{3}$$

This assumption is tantamount to the absence of pulse controls in the leg mobility degree, which could additionally “push” the body with a leg losing contact with the supporting surface.

Before the impact, the body was rotating around the S_{i-1} motionless foot and at the moment of the S_i foot contact with the supporting surface (before the impact) had the following angular speed:

$$\omega_i^- > 0, \tag{4}$$

then

$$\dot{x}_{Ci}^- = \omega_i^- l \cos\alpha, \quad \dot{y}_{Ci}^- = -\omega_i^- l \sin\alpha. \tag{5}$$

By virtue of theorems on the center of mass motion and system angular momentum alteration relative to the center of mass upon impact, let us write down:

$$\begin{gathered} m(\dot{x}_{Ci}^+ - \omega_i^- l \cos\alpha) = X_{i-1} + X_i; \\ m(\dot{y}_C^+ + \omega_i^- l \sin\alpha) = Y_{i-1} + Y_i; \\ m\rho^2(\omega_i^+ - \omega_i^-) = (Y_{i-1} - Y_i) l \sin\alpha - (X_{i-1} + X_i) l \cos\alpha. \end{gathered} \tag{6}$$

Two different types of impact are possible.

Type 1. As a result of the impact, both legs remain staying on the supporting surface. Shock reactions occur at the support points of both legs. After the impact, the wheel stops:

$$\dot{x}_{Ci}^+ = 0, \ \dot{y}_{Ci}^+ = 0, \ \omega_i^+ = 0. \tag{7}$$

Substituting (7) into (6), the following is obtained:

$$-m\omega_i^- \ l \cos\alpha = X_{i-1} + X_i;$$

$$m\omega_i^- \ l \sin\alpha = Y_{i-1} + Y_i;$$

$$-m\rho^2\omega_i^- = (Y_{i-1} - Y_i) l \sin\alpha - (X_{i-1} + X_i) l \cos\alpha.$$

Then

$$Y_{i-1} = -\frac{\rho^2 + l^2 \cos 2\alpha}{2l \sin\alpha} m\omega_i^-; \ Y_i = \frac{\rho^2 + l^2}{2l \sin\alpha} m\omega_i^-. \tag{8}$$

Condition of non-negativity of the vertical shock reaction components (1) due to (4) and (8) is equivale to the following condition:

$$\cos 2\alpha \le -\left(\frac{\rho}{l}\right)^2 < 0. \tag{9}$$

Type 2. After the impact, the body starts to rotate around the S_i motionless foot with the angular speed of

$$\omega_i^+ > 0, \ \dot{x}_{Ci}^+ = \omega_i^+ \ l \cos\alpha, \ \dot{y}_{Ci}^+ = \omega_i^+ l \sin\alpha. \tag{10}$$

Substituting (10) and (3) in (6), the following is obtained:

$$m(\omega_i^+ - \omega_i^-)\, l\cos\alpha = X_i;$$

$$m(\omega_i^+ + \omega_i^-)\, l\sin\alpha = Y_i;$$

$$m\rho^2(\omega_i^+ - \omega_i^-) = -Y_i l\sin\alpha - X_i l\cos\alpha.$$

Then

$$\omega_i^+ = k\omega_i^-;$$

$$Y_i = 2\frac{\rho^2 + l^2\cos^2\alpha}{\rho^2 + l^2} m\omega_i^- l\sin\alpha, \qquad (11)$$

where

$$k = \frac{\rho^2 + l^2\cos 2\alpha}{\rho^2 + l^2}. \qquad (12)$$

Rotation around the S_i foot results from the impact, when and only when $\omega_i^+ > 0,\ Y_i > 0.$ By virtue of (4), (10) and (11), these conditions are satisfied if $k > 0$ or

$$\cos 2\alpha > -\left(\frac{\rho}{l}\right)^2. \qquad (13)$$

It follows from (9) and (12) that the impact model is corrected (deterministic). For any parameter values, only one of the two possible types of impact is taking place. Let us note that the nature of wheel motion after the impact depends only on the design parameters and does not depend on the wheel angular speed before the impact.

Angle $\alpha = \pi / n,$ where n is the number of legs. With $n \geq 5,$ the 2α angle between adjacent legs is acute $\cos 2\alpha > 0,$ and condition (12) is always satisfied. At $n = 4,$ $\cos 2\alpha = 0$ and condition (12) are satisfied, if $\rho \neq 0.$ Let us note that $\rho = 0$ corresponds to the case, when the entire wheel mass is concentrated in its center of mass. For $n = 3$ $\cos 2\alpha = -1/2$ and condition (12) are satisfied, if $\rho > \sqrt{2}l.$

Let us consider further only the case, when condition (12) is satisfied, and, as a result of the impact, the S_{i-1} foot loses contact with the supporting surface, and rotation begins around the S_i, at the same time:

$$0 < k < 1. \qquad (14)$$

By virtue of (10), $0 < \omega_i^+ < \omega_i^-$, i.e., energy is lost upon the impact. With an increase in the number of legs, the $\alpha = \pi / n$ angle decreases, while the k

coefficient increases. In the limit at $n \to \infty$ the coefficient is $k \to 1$. Limiting transition to the infinite number of legs could be interpreted as transition to an ordinary wheel.

Rotation around the supporting leg. After the impact, the wheel rotates around the S_i foot (see Fig. 1 *b*). In accordance with the theorem on the kinetic moment alteration relative to point S_i:

$$(\rho^2 + l^2)\ddot{\varphi} = gl \sin\varphi, \tag{15}$$

where φ is the wheel rotation angle. Let us note that the wheel angular speed is $\omega = \dot{\varphi}$. At the beginning of this phase of motion, $\varphi_0 = \varphi_i^+ = \beta - \alpha$, $\omega_0 = \omega_i^+$. Equation (15) is equation of the inverted pendulum motion.

Equation (15) has the energy integral:

$$\frac{\omega^2}{2} + \frac{gl}{\rho^2 + l^2}\cos\varphi = \frac{(\omega_i^+)^2}{2} + \frac{gl}{\rho^2 + l^2}\cos(\beta - \alpha). \tag{16}$$

If at $\beta < \alpha$ the wheel reaches its critical position, when the center of mass is above the S_i support point (i.e., $\varphi = 0$) with the nonzero angular velocity, then it would pass over this position and collides with the supporting surface by the next foot S_{i+1}. For this, the following is necessary and sufficient due to the energy integral (16):

$$\omega_i^+ > \omega_{cr},$$

where

$$\omega_{cr} = \sqrt{\frac{2gl}{\rho^2 + l^2}\big(1 - \cos(\beta - \alpha)\big)}. \tag{17}$$

If $\beta = \alpha$, then $\omega_{cr} = 0$.

If $\beta > \alpha$, wheel position resting on the S_{i-1} and S_i legs is statically unstable, and under the gravity force action at any $\omega_i^+ \geq 0$ values the wheel rotates around the S_i, foot, then the wheel will continue to move down the slope and collides with it by the next leg S_{i+1}. In this case, the wheel would begin to move down the slope even from the rest state.

Walking wheel motion non-separation condition from the supporting surface. When the wheel moves, the constrain at the leg support point is unilateral. The supporting leg may lose contact with the supporting surface at high values of the wheel angular speed. In this case, the wheel passes from “walking” to “running”. This happens, when reaction normal to the supporting surface required to ensure the wheel rotation, is less than zero $(Y_i < 0)$. Let us consider here it to be unacceptable; i.e., our research would be restricted to studying the wheel “walking” mode. For this, it is necessary and sufficient that at each step $Y_i > 0$.

In accordance with the theorem on the center of mass motion:

$$Y_i = m(g\cos\beta + \ddot{y}_C) = m(g\cos\beta - l\cos(\varphi-\beta)\dot{\varphi}^2 - l\sin(\varphi-\beta)\ddot{\varphi}). \qquad (18)$$

Substituting the $\ddot{\varphi}$ relation from (15) and $\dot{\varphi}^2 = \omega^2$ from (16) into (18), wheel motion non-separation condition is obtained at the i-th step:

$$\Phi(\omega_i^+,\ \beta, \alpha) > 0, \qquad (19)$$

where

$$\Phi(\omega_i^+, \beta, \alpha) =$$

$$= \min_{\varphi\in[\beta-\alpha,\ \beta+\alpha]} \left\{ g\cos\beta - \frac{gl^2}{\rho^2 + l^2}[2\cos(\varphi-\beta)[\cos(\beta-\alpha) - \cos\varphi] - \right.$$

$$\left. - l\sin(\varphi-\beta)\sin\varphi] - l\cos(\varphi-\beta)(\omega_i^+)^2 \right\}.$$

Poincaré map. Self-oscillations. If $\omega_i^+ < \omega_{cr}$, the wheel is not reaching its critical position, and under the action of gravity starts to rotate in the opposite direction and collides with the supporting surface by the preceding leg S_{i-1}. At the repeated collision with the S_{i-1} leg on the supporting surface and due to the energy integral, $\omega_{i+1}^- = -\omega_i^+$, then after the impact:

$$\omega_{i+1}^+ = f_1(\omega_i^+) = -k\omega_i^+. \qquad (20)$$

Relation (20) is the Poincaré map for the angular speed alteration per i-th step. It is linear and has a single fixed point $\omega_0 = 0$, which corresponds to stable equilibrium position with support on the S_{i-1} and S_i two legs by virtue of the Koenig’s theorem [20], since:

$$\left| \frac{df_1}{d\omega_i^+} \right| = k < 1.$$

The k value is determined by relationship (12) and $0 < k < 1$.

Poincaré map (20) is presented in Fig. 3 *a*. Angular speed is decreasing exponentially. It could be shown that each step duration also decreases in geometric progression. An endless series of collisions is taking place accompanied by waddling from one leg to the other, but total duration of this series of collisions is finite.

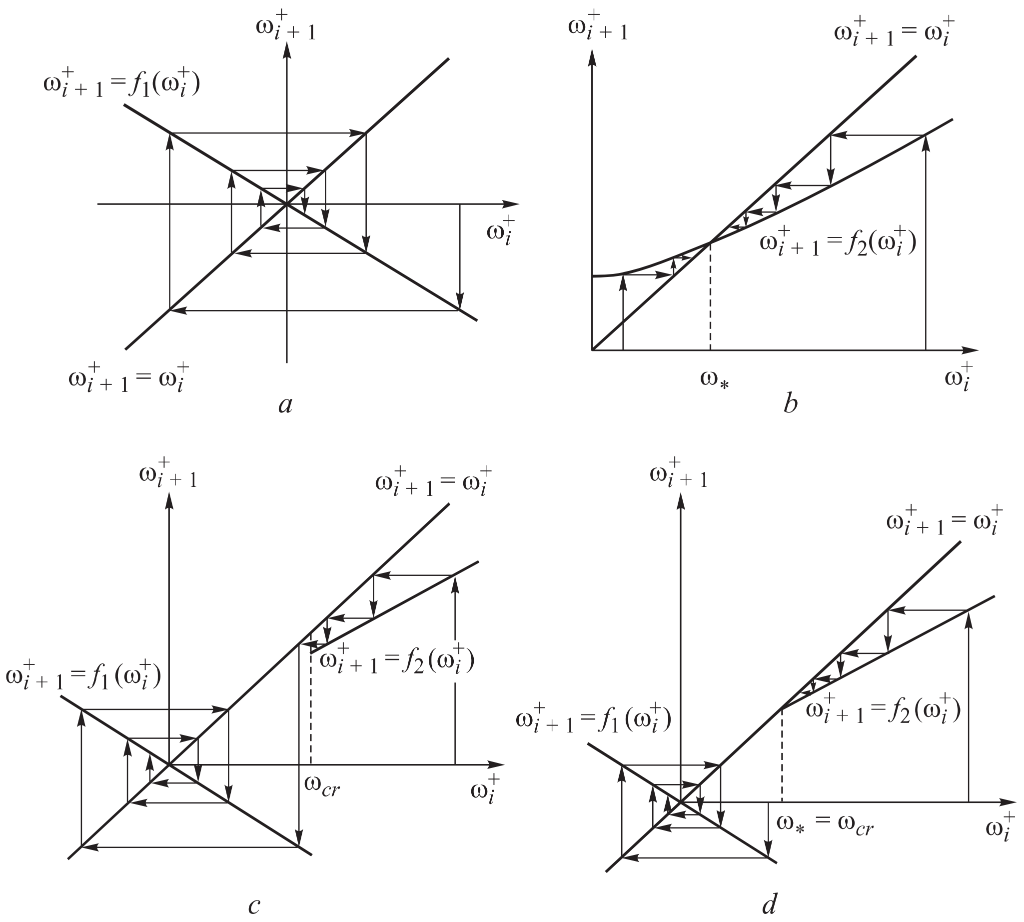

Fig. 3. Poincaré map at $\beta < \alpha$ and $\omega_1^{+} < \omega_{cr}$ (*a*); at $\beta \geq \alpha$ or $\beta < \alpha$, $\omega_1^{+} > \omega_{cr}$ and $\omega_* > \omega_{cr}$ (*b*); at $\beta < \alpha$, $\omega_1^{+} > \omega_{cr}$ and $\omega_* < \omega_{cr}$ (*c*); at $\beta < \alpha$ and $\omega_* = \omega_{cr}$ (*d*)

If $\omega_i^{+} = \omega_{cr}$, the wheel would reach its critical position for the infinitely long time.

If $\beta \geq \alpha$ or $\beta < \alpha$ and $\omega_i^{+} > \omega_{cr}$, then the wheel turns over the supporting leg. The energy integral (16) makes it possible to determine the wheel angular speed at the S_{i+1} foot collision with the supporting surface. Taking into account that in this position $\theta = \theta_{i+1}^{-} = \beta + \alpha$, the following is obtained:

$$(\omega_{i+1}^{-})^2 = (\omega_i^{+})^2 + \frac{2gl}{\rho^2 + l^2}\left[\cos(\beta - \alpha) - \cos(\beta + \alpha)\right] =$$

$$= (\omega_i^{+})^2 + \frac{4gl}{\rho^2 + l^2}\sin\alpha\sin\beta, \qquad (21)$$

where $\omega_{i+1}^{-} > 0$ is the wheel angular speed at the end of the i-th step or the same at the beginning of the (i + 1)-th step.

From (11) for the impact with the S_{i+1} foot and from (21), the Poincaré map is obtained for the wheel angular speed alteration in the i-th step, which consists of the rotation stage around the S_i foot and the following impact by foot S_{i+1}:

$$\omega_{i+1}^{+} = f(\omega_i^{+}) = k\left[(\omega_i^{+})^2 + \frac{4gl}{\rho^2 + l^2}\sin\alpha\sin\beta\right]^{1/2}. \qquad (22)$$

Map (22) has a fixed point:

$$\omega_{*} = \left[\frac{k^2}{1-k^2}\frac{4gl}{\rho^2 + l^2}\sin\alpha\sin\beta\right]^{1/2}.$$

In accordance with the Koenig's theorem [20], this fixed point corresponds to a stable limiting cycle, since

$$\left|\frac{df}{d\omega_i^{+}}\right| = k^2 < 1 \text{ at } \omega = \omega_{*}.$$

However, the wheel actually reaches this stable periodic solution only at $\beta \geq \alpha$ or $\beta < \alpha$, $\omega_i^{+} > \omega_{cr}$ and $\omega_{*} > \omega_{cr}$. Poincaré map (22) is presented in Fig. 3 *b*.

Let us note that stable periodic mode of motion takes place, if at each step the condition of the wheel motion non-separation from the supporting surface is observed. For this, it is necessary and sufficient that this condition should be satisfied at the limiting cycle and at the first step, i.e.,

$$\Phi(\omega_{*}, \beta, \alpha) > 0, \quad \Phi(\omega_1^{+}, \beta, \alpha) > 0.$$

At $\beta < \alpha$, $\omega_i^{+} > \omega_{cr}$ and $\omega_{*} < \omega_{cr}$, there is no periodic solution corresponding to fixed point ω_{*}. This is explained by the fact that the Poincaré map (22) is valid only as long as the wheel turns over the supporting leg. Over the course of several steps, the wheel angular speed would decrease until it becomes less than ω_{cr}. After that, the wheel would waddle from one leg on the other in accordance with the Poincaré map (20). For this case, the Poincaré map is presented in Fig. 3 *c*.

At $\beta < \alpha$, $\omega_i^{+} > \omega_{cr}$ and $\omega_{*} = \omega_{cr}$, the wheel would tend to a periodic solution corresponding to fixed point ω_{*}. However, this periodic solution is not stable, since the Poincaré map takes place at $\omega_i^{+} < \omega_{cr}$ (20). For this case, the Poincaré map is presented in Fig. 3 *d*.

Conclusion. Problem of the walking wheel motion down an inclined plane in nonlinear formulation was analytically investigated. The walking wheel is the simplest model of passive bipedal walking. Probable cases of the walking wheel movement were investigated for different values of the supporting surface inclination and of the wheel initial angular speed. It is shown that different modes of the walking wheel movement are possible. Existence of the stable periodic solution (self-oscillations) appears to be the most interesting solution.

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Lapshin V.V. — Dr. Sc. (Phys.-Math.), Professor, Department of Theoretical Mechanics, Bauman Moscow State Technical University (2-ya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russian Federation).

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