

STUDYING OPTICAL CHARACTERISTICS OF DIFFUSED LIGHT REFLECTING FROM NATURALLY SENESCING LEAVES OF DECIDUOUS TREES

Yu.V. Mamelin

ymamelin@bk.ru

G.F. Kopytov

g137@mail.ru

V.Yu. Buzko

buzkonmr@mail.ru

**Federal State Budgetary Educational Institution of Higher Education
“Kuban State University”, Krasnodar, Russian Federation**

Abstract

Study object included leaves of deciduous trees in the Krasnodar Territory at different stages of senescing visually manifested in their color alteration. Study subject was the optical characteristics of light diffused reflection from green, yellow-green and yellow leaves of deciduous trees in the Krasnodar Territory during the autumn season. Work objective lies in identifying the possibility to establish differences between green leaves of deciduous trees, and yellow-green and yellow leaves of deciduous trees using the terrain multispectral and hyperspectral sounding methods, as well as collecting information on spectral characteristics of the diffused light being reflected from various biological objects. Results of quantitative and qualitative analysis of data obtained through the diffused light reflectance spectroscopy from leaves of deciduous trees are presented. Narrow-band vegetation indices $mNDVI_{705}$, mSR_{705} , $CRI1$, $SIPI$ and $PSRI$ were used in quantitative analysis of data on the diffused light reflection spectra obtained from green, yellow-green and yellow leaves of deciduous trees. It was revealed that the use of narrow-band vegetation indices in the remote sensing algorithms using multi- and hyperspectral cameras makes it possible to rather accurately distinguish leaves at different stages of senescing. Optical characteristics of diffused light reflection from green, yellow-green and yellow leaves of deciduous trees, which are typical species of trees in urban and rural plantings in the Krasnodar Territory, are described for the first time

Keywords

Spectroscopy, diffused reflection, vegetation indices, leaves of deciduous trees, chlorophylls, carotenoids, $NDVI_{705}$ index, $CRI1$ index

Received 21.11.2019

Accepted 23.12.2019

© Author(s), 2020

Introduction. Seasonal changes in the color of tree leaves depend on such external factors as decrease in the sunny day duration, total daily temperature and level of solar radiation [1]. Color of tree leaves also directly depends on the internal biochemical processes that are connected to regulation of the nutrient substances' resorption from leaf cells often under conditions of biotic and abiotic stress [2–6]. This leads to leaf senescing within the phenological cycle, which results in chloroplasts senescing and turning them into chromoplasts. Chlorophylls contained in leaf cells are destroyed during senescing; brightly colored carotenoids and anthocyanins are being accumulated at the chlorophylls' positions [7–13]. Studying and accumulating data on the spectra of diffused light reflection (DLR) from the degrading leaves of vigorous deciduous trees makes it possible to solve several urgent problems of remote monitoring the forest and agroindustrial territories [14–16], as well as the urban green spaces in order to distinguish vigorous plants from the diseased [17, 18]. Study of the optical properties of DLR from leaves of vigorous deciduous trees during the autumn season would make it possible to monitor dynamics of alterations in the amount of pigment content in the leaf structure depending on their color and senescing.

The purpose of the work is to identify a possibility of distinguishing green leaves of deciduous trees from yellow-green and yellow leaves of deciduous trees using the multispectral and hyperspectral sounding methods, as well as to accumulate information on spectral characteristics of the DLR from the naturally senescing leaves of deciduous trees in the Krasnodar Territory in the wavelength range of 350–900 nm. Based on the objective set, the following tasks should be fulfilled.

Studying optical characteristics of the DLR from green, yellow-green and yellow leaves of vigorous deciduous trees, typical for regions with a transitional climate from temperate continental to subtropical, collected in the territory of the city of Krasnodar.

Conducting qualitative and quantitative analysis of the obtained DLR spectra from green, yellow-green and yellow leaves of deciduous trees in the Krasnodar Territory.

Based on the analysis conducted, drawing conclusions about the possibility to distinguish the mass of the deciduous trees green leaves and yellow leaves by remote sensing of the terrain using multispectral and hyperspectral cameras.

Materials and equipment. To conduct the research, samples of green, yellow-green and yellow leaves of vigorous deciduous trees were collected in the territory of the Kuban State University Botanical Garden in September — October 2018–2019 including walnut (*Juglans regia*), apricot (*Prunus armeniaca*), cherry (*Prunus cerasus*), ash (*Fraxinus*), oak (*Quercus*) and maple (*Acer*).

The listed representatives are typical species of trees in the populated territory of the Krasnodar Territory, in urban and rural landscapes. Samples of selected deciduous trees' leaves were cut from lower and middle parts of the tree crown and included six samples of the same species, three of which were from the crown shadow part and three from the crown lighted part.

DLR spectra from the selected tree leaves' samples were measured for 1.5 h after they were cut from tree branches in laboratory conditions using the *Hitachi U3900* spectrophotometer with a two-channel integrating sphere. Schematic diagram of the spectrophotometer operation is presented in Fig. 1. Preliminary experiments demonstrated that the DLR spectra of the collected leaves stored in a dark container were not changing in the time range of 0.5–6.0 h before measurement.

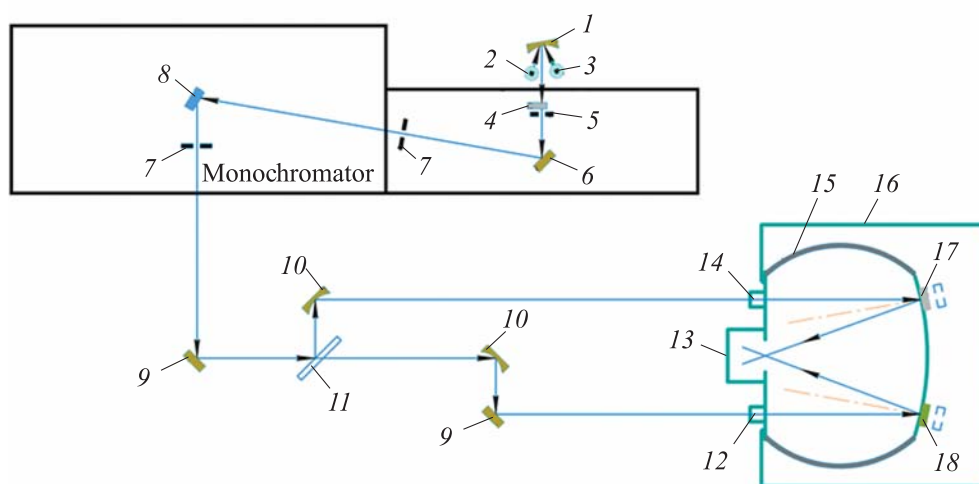


Fig. 1. Schematic operation diagram of the *Hitachi U3900* spectrophotometer with a two-channel integrating sphere:

1 is collecting mirror; 2 is wolfram (tungsten)-iodine (WI) lamp; 3 is deuterium (D₂) lamp; 4 is filter; 5 is slit; 6 is spherical mirror; 7 is exit slit; 8 is stigmatic diffraction grating with variable pitch; 9 is flat mirror; 10 is toroidal mirror; 11 is rotating mirror; 12 is measurement channel; 13 is light trap (photodetector); 14 is comparison channel; 15 is integrating sphere; 16 is locking hatch; 17 is reflection standard; 18 is test sample

Wolfram (tungsten)-iodine (WI) lamp is used as a light source in the spectrophotometer in the visible and near IR region of 370–1100 nm, and deuterium (D₂) lamp is used in the ultraviolet range of 190–370 nm. To ensure stable measurements, two-beam optics are introduced, where the monochromatic beam is selected by a monochromator and is divided into a reference beam and a test beam with the rotating mirror (sector). These beams are

directed to a closed compartment with a two-channel integrating sphere for samples (measured sample and reflection standard in the form of a ceramic tablet made of ultra-high-purity magnesium oxide MgO).

DLR spectra from the selected tree leaf samples were measured within the spectral wavelength range of 350–900 nm with resolution of 1 nm. Three DLR spectra were obtained for the experiment purity from different regions of each sample under study, and then the arrays of these DLR spectra were averaged within a single sample of the biological species under study.

Measurement and discussion results. Obtained averaged results on the mass array of the DLR spectra from senescing leaves of deciduous trees are presented in Fig. 2. DLR spectra from green, yellow-green and yellow leaves

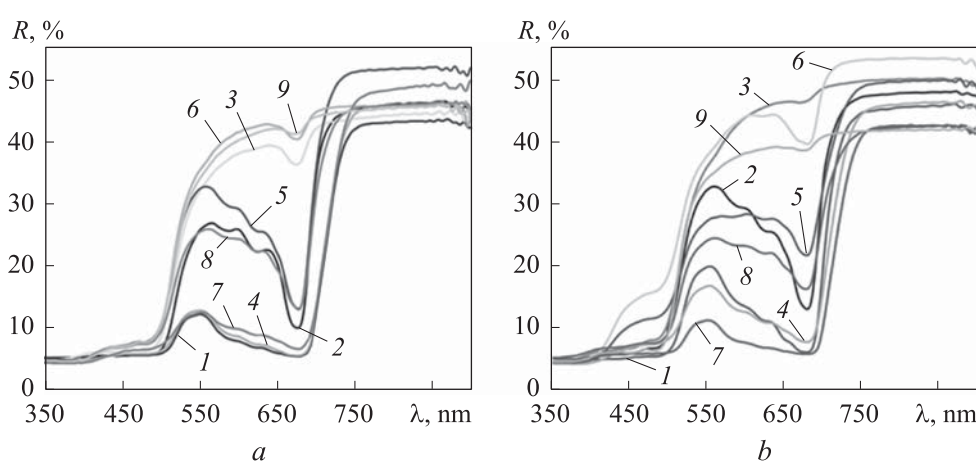


Fig. 2. Optical spectra of DLR from green, yellow-green and yellow leaves:

a fruit trees (walnut, *Juglans regia*): 1 green, 2 yellow-green, 3 yellow;
 apricot (*Prunus armeniaca*): 4 green, 5 yellow-green, 6 yellow; cherry (*Prunus cerasus*):
 7 green, 8 yellow-green, 9 yellow; *b* deciduous trees: ash (*Fraxinus*): 1 green, 2 yellow-green,
 3 yellow; oak (*Quercus*): 4 green, 5 yellow-green, 6 yellow; maple (*Acer*): 7 green,
 8 yellow-green, 9 yellow

within the frames of a single species are significantly different. It should be noted that the DLR spectra of leaves of the same coloring are not much differentiating. In the range 350–500 nm, all the leaf samples under study are manifesting some low reflectance coefficient (RC) of approximately 5 %. This is not surprising, since the chlorophyll-*a* and chlorophyll-*b* absorption peaks are respectively appearing at wavelengths of 440 and 460 nm [9, 11, 12, 19–22]. Rapid increase in reflectivity of the leaf samples under study is observed in the range of 490–520 nm values [11]. Maximum RC values in the spectrum visible region for green leaf samples is 11–13 % at wavelengths of not more than

560 nm, for yellow-green leaf samples — 25–35 % at wavelengths of about 560 nm, for yellow leaf samples — 45–50 % at wavelengths of about 630 nm. This is strongly/satisfactory agreed with theoretical data [11]. Upon reaching the maximum RC values in the spectrum visible region, spectral characteristics of the leaf samples under study are gradually decreasing and are reaching a local minimum in around 670 nm, which is determined by the chlorophyll ability to absorb light in this spectrum region [9–11, 17, 20, 23]. RC value for green leaf samples in this region is approximately 5 %, for yellow-green leaf samples — 10–15 %, and for yellow leaf samples — 32–42 %. After passing the local minimum point, RC for all the leaf samples under study is rapidly increasing by an average of 45–47 %, and then forms a plateau in the near IR wavelength range.

Quantitative analysis results of the DLR spectra from the green, yellow-green, and yellow leaves of deciduous trees under study showed that RC values in the visible spectral region of 550–700 nm are significantly differing in their regard. Thus, when considering the reflection spectra for all the leaf samples under study relative to the green samples' spectra at a wavelength of 550 nm, then the green leaf samples RC value is by 60 % lower than that of the yellow-green and 105 % lower than that of the yellow leaves. If DLR spectra are considered relative to the light reflection local minimum point in the region of about 680 nm typical for green leaves, then the green leaf samples RC value is by 46 % lower than that of the yellow-green and by 185 % lower than that of the yellow leaves.

To carry out the qualitative analysis, let us use vegetation indices (VI) effectively used in precision farming [24–26], and, namely, those indices characterizing stress in leaves, alterations in the chlorophylls and carotenoids content in the narrow spectral wavelength ranges:

- 1) SR modified index at the boundary of the spectrum red region:

$$mSR_{705} = \frac{p_{750} - p_{445}}{p_{705} - p_{445}};$$

- 2) modified normalized difference index at the spectrum red region:

$$mNDVI_{705} = \frac{p_{750} - p_{705}}{p_{750} + p_{705} - 2p_{445}};$$

- 3) photochemical reflection index:

$$PSRI = \frac{p_{680} - p_{500}}{p_{700}};$$

4) index insensitive to the structure pigments:

$$SIPI = \frac{p_{800} - p_{445}}{p_{800} + p_{680}};$$

5) carotenoid reflection index:

$$CRI1 = \frac{1}{p_{510}} - \frac{1}{p_{550}}.$$

Here p_{xxx} is the estimate based on reflectance at the corresponding wavelength (xxx is the wavelength, nm).

VI values calculated according to the given formulas in regard to samples under study of deciduous trees' senescing leaves are presented in the following Table.

VI values of the deciduous trees' leaves during the autumn season

No.	Coloring	mSR_{705}	$mNDVI_{705}$	$PSRI$	$SIPI$	$CRI1$
<i>Walnut (Juglans regia)</i>						
1	Green	3.27	0.36	-0.005	1.00	6.91
2	Yellow-green	1.26	0.09	0.116	1.17	7.42
3	Yellow	1.04	0.01	0.665	5.36	4.79
<i>Apricot (Prunus armeniaca)</i>						
4	Green	3.59	0.39	-0.014	1.00	6.02
5	Yellow-green	1.38	0.12	0.085	1.20	3.63
6	Yellow	1.02	0.01	0.685	8.66	3.32
<i>Cherry (Prunus cerasus)</i>						
7	Green	3.69	0.39	-0.005	1.02	4.31
8	Yellow-green	1.41	0.12	0.113	1.23	4.14
9	Yellow	1.02	0.01	0.708	10.56	3.71
<i>Ash-tree (Fraxinus)</i>						
10	Green	1.79	0.24	0.007	1.04	6.73
11	Yellow-green	1.29	0.11	0.092	1.25	3.53
12	Yellow	1.02	0.01	0.723	12.42	3.01
<i>Oak (Quercus)</i>						
13	Green	2.48	0.34	-0.003	1.04	4.34
14	Yellow-green	1.48	0.15	0.191	1.47	2.46
15	Yellow	1.07	0.03	0.408	4.49	1.71
<i>Maple (Acer)</i>						
16	Green	3.64	0.45	-0.004	1.00	6.16
17	Yellow-green	1.39	0.13	0.192	1.40	3.62
18	Yellow	1.02	0.01	0.718	11.38	3.23

The mSR_{705} and $mNDVI_{705}$ VI obtained values [9, 13, 24] provide a comprehensive assessment of the state of the leaf samples under study. From them it could be concluded that samples of green leaves under study are characterized by chloroplast degradation, destruction of chlorophylls, and disclosure of carotenoids and anthocyanins. Destruction of chlorophylls in the yellow leaf samples occurred completely. As a result, lignin and cellulose concentration increased, which was confirmed by the $PSRI$ values [11, 26]. Regardless of changes in the leaf structure, $SIPI$ VI [13, 24, 26] reflects the ratio of bulk carotenoids to chlorophyll; $SIPI$ value in the studied samples of yellow leaves is by 5–10 times higher than the values in green and yellow-green leaves. While analyzing the $CRII$ VI values [10, 13], it is impossible to unequivocally reveal dependence of the carotenoid content on the leaf coloring. However, it could be stated that $mNDVI_{705}$, mSR_{705} , $SIPI$, $PSRI$ VIs and their various combinations could be introduced in machine recognition of deciduous trees leaf coloring during the autumn season and not only using the remote means of multispectral and hyperspectral terrain scanning.

Diagram of the area distribution under the spectra graphs of samples under study in the wavelength range of 500–700 nm is provided in Fig. 3 for visual comparison of the naturally senescing leaf samples under study in the visible

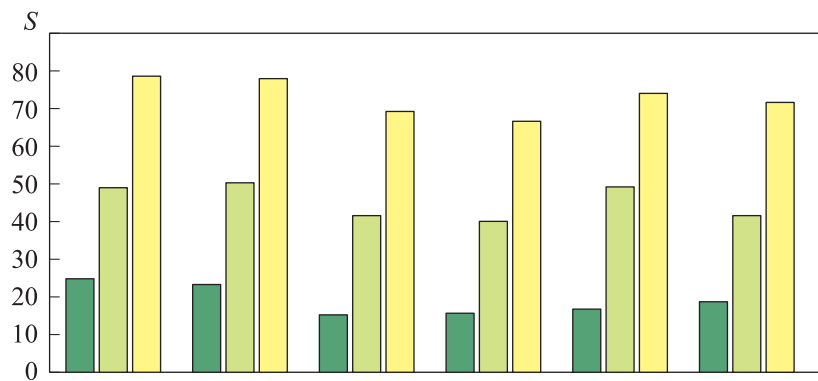


Fig. 3. Area distribution diagram under the spectra graphs of samples under study in the spectral range of 500–700 nm:

■ green leaves spectra values; ■ yellow-green leaves spectra values;
 ■ yellow leaves spectra values

spectrum region. With the altering leaf color caused by degradation of chlorophylls, there appears an increase in the sub-spectral areas of the deciduous trees' yellow-green and yellow leaf samples in regard to the green leaves' spectra sub-spectral areas by approximately 1.8–2.2 and 3.2–4.5 times, respectively.

Further research stage would require to check feasibility of using the listed VIs for the dried green, yellow-green and yellow leaves of deciduous trees of the Krasnodar Territory.

Conclusion. Data on the DLR optical characteristics in the wavelength range of 350–900 nm from green, yellow-green and yellow leaves of deciduous trees growing in the southern territories of the Russian Federation and selected during the autumn season were accumulated. Qualitative and quantitative analysis of the data obtained was carried out. Based on the qualitative analysis results, conclusion could be made that spectral characteristics of DLR from senescing leaves are having the RC maximum peaks at different wavelengths. This makes it possible to distinguish them from each other by the type of spectral characteristic. Quantitative analysis results of studying the deciduous leaf samples showed that the determination of the leaves' green level and of the carotenoids content is possible by the leaves' RC value in the narrow wavelength ranges. Based on qualitative and quantitative analysis of the DLR spectra from green, yellow-green and yellow leaves, a conclusion could be made on practical possibility to create a machine algorithm for recognizing leaf color on the basis of VI for remote multispectral terrain sounding systems, or using the reflected light registration methods by CCD or CMOS matrices, provided that the area under study is illuminated by laser LEDs of a certain wavelength or of several wavelengths.

Translated by K. Zykova

REFERENCES

- [1] Liu Q., Fu Yo., Zeng Z., et al. Temperature, precipitation, and insolation effects on autumn vegetation phenology in temperate China. *Glob. Chang. Biol.*, 2016, vol. 22, iss. 2, pp. 644–655. DOI: <https://doi.org/10.1111/gcb.13081>
- [2] Ougham H.J., Morris Ph., Thomas H. The colors of autumn leaves as symptoms of cellular recycling and defenses against environmental stresses. *Curr. Top. Dev. Biol.*, 2005, vol. 66, pp. 135–130. DOI: [https://doi.org/10.1016/S0070-2153\(05\)66004-8](https://doi.org/10.1016/S0070-2153(05)66004-8)
- [3] Pollet B., Steppe K., Dambre P., et al. Seasonal variation of photosynthesis and photosynthetic efficiency in *Phalaenopsis*. *Photosynthetica*, 2010, vol. 48, iss. 4, pp. 580–588. DOI: <https://doi.org/10.1007/s11099-010-0075-7>
- [4] Broeckx L.S., Fichot R., Verlinden M.S., et al. Seasonal variations in photosynthesis, intrinsic water-use efficiency and stable isotope composition of poplar leaves in a short-rotation plantation. *Tree Physiol.*, 2014, vol. 34, iss. 7, pp. 701–715. DOI: <https://doi.org/10.1093/treephys/tpu057>
- [5] Weng J.-H., Liao T.-S., Sung K.-H., et al. Seasonal variations in photosynthesis of *Picea morrisonicola* growing in the subalpine region of subtropical Taiwan. *Tree Physiol.*, 2005, vol. 25, iss. 8, pp. 973–979. DOI: <https://doi.org/10.1093/treephys/25.8.973>

- [6] Robledo D., Freile-Pelegrin Y. Seasonal variation in photosynthesis and biochemical composition of *Caulerpa* spp. (Bryopsidales, Chlorophyta) from the Gulf of Mexico. *Phycologia*, 2005, vol. 44, no. 3, pp. 312–319.
- [7] Lee D.W., O’Keefe J., Holbrook N.M., et al. Pigment dynamics and autumn leaf senescence in a New England deciduous forest, eastern USA. *Ecol. Res.*, 2003, vol. 18, iss. 6, pp. 677–694. DOI: <https://doi.org/10.1111/j.1440-1703.2003.00588.x>
- [8] Liu N., Lin Z.-F., Van Devender A., et al. Spectral reflectance indices and pigment functions during leaf ontogenesis in six subtropical landscape plants. *Plant Growth Regul.*, 2008, vol. 58, no. 1, pp. 73–84. DOI: <https://doi.org/10.1007/s10725-008-9353-9>
- [9] Sims D.A., Gamon J.A. Relationships between leaf pigment content and spectral reflectance across a wide range of species, leaf structures and developmental stages. *Remote Sens. Environ.*, 2002, vol. 81, iss. 2-3, pp. 337–354. DOI: [https://doi.org/10.1016/s0034-4257\(02\)00010-x](https://doi.org/10.1016/s0034-4257(02)00010-x)
- [10] Gitelson A.A., Zur Y., Chivkunova O.B., et al. Assessing carotenoid content in plant leaves with reflectance spectroscopy. *Photochem. Photobiol.*, 2002, vol. 75, iss. 3, pp. 272–281. DOI: [https://doi.org/10.1562/0031-8655\(2002\)0750272ACCIPL2.0.CO2](https://doi.org/10.1562/0031-8655(2002)0750272ACCIPL2.0.CO2)
- [11] Merzlyak M.N., Gitelson A.A., Chivkunova O.B., et al. Non-destructive optical detection of pigment changes during leaf senescence and fruit ripening. *Physiol. Plant.*, 1999, vol. 106, iss. 1, pp. 135–141. DOI: <https://doi.org/10.1034/j.1399-3054.1999.106119.x>
- [12] Merzlyak M.N., Chivkunova O.B., Solovchenko A.E., et al. Light absorption by anthocyanins in juvenile, stressed, and senescing leaves. *J. Exp. Bot.*, 2008, vol. 59, iss. 14, pp. 3903–3911. DOI: <https://doi.org/10.1093/jxb/ern230>
- [13] Zhou X., Huang W., Zhang J., et al. A novel combined spectral index for estimating the ratio of carotenoid to chlorophyll content to monitor crop physiological and phenological status. *Int. J. Appl. Earth Obs. Geoinf.*, 2019, vol. 76, pp. 128–142. DOI: <https://doi.org/10.1016/j.jag.2018.10.012>
- [14] Hovi A., Raitio P., Rautiainen M. A spectral analysis of 25 boreal tree species. *Silva Fennica*, 2017, vol. 51, no. 4, art. 7753. DOI: <https://doi.org/10.14214/sf.7753>
- [15] Möttus M., Sulev M., Hallik L. Seasonal course of the spectral properties of alder and birch leaves. *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.*, 2014, vol. 7, iss. 6, pp. 2496–2505. DOI: <https://doi.org/10.1109/jstars.2013.2294242>
- [16] Forsström P., Peltoniemi J., Rautiainen M. Seasonal dynamics of lingonberry and blueberry spectra. *Silva Fennica*, 2019, vol. 53, no. 2, art. 10150. DOI: <https://doi.org/10.14214/sf.10150>
- [17] Zhang X., Han L., Dong Y., et al. A deep learning-based approach for automated yellow rust disease detection from high-resolution hyperspectral UAV Images. *Remote Sens.*, 2019, vol. 11, iss. 13, art. 1554. DOI: <https://doi.org/10.3390/rs11131554>
- [18] Randive P.U., Deshmukh R.P., Janse P.V., et al. Discrimination between healthy and diseased cotton plant by using hyperspectral reflectance data. In: Santosh K., Hega-

di R. (eds). *Recent Trends in Image Processing and Pattern Recognition. RTIP2R 2018. Communications in Computer and Information Science*, vol. 1037. Singapore, Springer, pp. 342–351. DOI: https://doi.org/10.1007/978-981-13-9187-3_30

[19] Lee Z., Carder K., Arnone R., et al. Determination of primary spectral bands for remote sensing of aquatic environments. *Sensors*, 2007, vol. 7, iss. 12, pp. 3428–3441. DOI: <https://doi.org/10.3390/s7123428>

[20] Molkov A.A., Korchemkina E.N., Kalinskaya D.V., et al. Spatial and temporal variability of reflectance coefficient of Gorky reservoir waters: results of 2016–2017 expeditions. *Sovr. Probl. DZZ kosm.* [Current Problems in Remote Sensing of the Earth from Space], 2018, vol. 15, no. 2, pp. 201–212 (in Russ.). DOI: <https://doi.org/10.21046/2070-7401-2018-15-2-201-210>

[21] Menon H.B., Adhikari A. Remote sensing of chlorophyll-a in case II waters: a novel approach with improved accuracy over widely implemented turbid water indices. *J. Geophys. Res. Oceans.*, 2018, vol. 123, iss. 11, pp. 8138–8158. DOI: <https://doi.org/10.1029/2018jc014052>

[22] Kume A., Akitsu T., Nasahara K.N. Correction to: why is chlorophyll *b* only used in light-harvesting systems? *J. Plant Res.*, 2018, vol. 131, no. 6, pp. 961–972. DOI: <https://doi.org/10.1007/s10265-018-1052-7>

[23] Kanash E.V. [Main properties of agrophytocenosis for decoding spectral data of remote sounding]. *Mater. Vseros. nauch. konf. "Primenenie sredstv distantsionnogo zondirovaniya Zemli v sel'skom khozyaystve"* [Proc. Rus. Int. Conf. "Application of remote Earth sensing in agriculture"]. St. Petersburg, FGBNU AFI Publ., 2015, pp. 25–28 (in Russ.).

[24] Feret J.-B., Asner G.P. Tree species discrimination in tropical forests using airborne imaging spectroscopy. *IEEE Trans. Geosci. Remote Sens.*, 2013, vol. 51, iss. 1, pp. 73–84. DOI: <https://doi.org/10.1109/tgrs.2012.2199323>

[25] Peñuelas J., Filella I., Gamon J.A., et al. Assessing photosynthetic radiation-use efficiency of emergent aquatic vegetation from spectral reflectance. *Aquat. Bot.*, 1997, vol. 58, iss. 3-4, pp. 307–315. DOI: [https://doi.org/10.1016/s0304-3770\(97\)00042-9](https://doi.org/10.1016/s0304-3770(97)00042-9)

[26] Xue J., Su B. Significant remote sensing vegetation indices: a review of developments and applications. *J. Sens.*, 2017, vol. 2017, art. 1353691. DOI: <https://doi.org/10.1155/2017/1353691>

Mamelin Yu.V. — Junior Researcher, Laboratory of Robotics, Federal State Budgetary Educational Institution of Higher Education "Kuban State University" (Stavropolskaya ul. 149, Krasnodar, 350040 Russian Federation).

Kopytov G.F. — Dr. Sc. (Phys.-Math.), Professor, Head of the Department of Radiophysics and Nanotechnology, Federal State Budgetary Educational Institution of Higher Education "Kuban State University" (Stavropolskaya ul. 149, Krasnodar, 350040 Russian Federation).

Buzko V.Yu. — Cand. Sc. (Chem.), Assoc. Professor, Department of Radiophysics and Nanotechnology, Federal State Budgetary Educational Institution of Higher Education “Kuban State University” (Stavropolskaya ul. 149, Krasnodar, 350040 Russian Federation).

Please cite this article as:

Mamelin Yu.V., Kopytov G.F., Buzko V.Yu. Studying optical characteristics of diffused light reflecting from naturally senescing leaves of deciduous trees. *Herald of the Bauman Moscow State Technical University, Series Natural Sciences*, 2020, no. 5 (92), pp. 72–82. DOI: <https://doi.org/10.18698/1812-3368-2020-5-72-82>

В Издательстве МГТУ им. Н.Э. Баумана
вышла в свет монография авторов
М.П. Галанина, Е.Б. Савенкова

**«Методы численного анализа
математических моделей»**

Изложены методы решения задач линейной алгебры, систем нелинейных алгебраических уравнений, интерполяция функций, методы численного интегрирования и дифференцирования, численные методы решения задачи Коши и краевых задач для систем обыкновенных дифференциальных уравнений. Приведены основы общей теории разностных схем и ее применение к построению и анализу методов численного решения эллиптических, параболических и гиперболических уравнений, а также численные методы решения интегральных уравнений. Представлены методы генерации сеток для многомерных задач математической физики, многосеточные методы решения, численные методы для решения уравнения переноса и уравнений газовой динамики, алгоритмические основы метода конечных элементов.

По вопросам приобретения обращайтесь:
105005, Москва, 2-я Бауманская ул., д. 5, стр. 1
+7 (499) 263-60-45
press@bmstu.ru
<https://bmstu.press>

