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MOTION OF A CHARGED PARTICLE IN THE ELECTROMAGNETIC FIELD OF A POLARIZATION-MODULATED WAVE IN THE PRESENCE OF A CONSTANT MAGNETIC FIELD

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Abstract

This article continues the study of the motion and radiation of a charged particle in the field of a high-intensity polarization-modulated wave already in the presence of an external constant magnetic field. Formulas for the average kinetic energy of a particle are obtained without considering the rest energy in the case of circular and linear polarization of a modulated electromagnetic wave. The peculiarity of the energy characteristics of a charged particle was demonstrated on the graph of the dependence of the average kinetic energy on the magnitude of the external magnetic field. The solution of the equation of motion of a charged particle in a given combination of fields is of interest in studies of the interaction between laser radiation and plasma, in the development of multifrequency lasers and in laser modulation technology

Keywords

Plane electromagnetic wave, polarization modulation, charged particle, magnetic field, high-power laser radiation

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Introduction. Over the past 60 years, rapid development in the field of laser physics has allowed to introduce a number of innovations into nonlinear optics, including frequency transformation, ultra-suffering optics and completely optical modulation. Record intensities in the focus of a laser beam were achieved ~ 10^{23} W/cm^2 [1, 2]. The solution to the problem of managing such laser impulses is of great interest, but for this it is necessary to overcome a number of difficulties [3]. In [4, 5], the authors using plasma modulators changing the polarization of radiation control laser impulse. The polarization modulation, which creates a two-dimensional signal, unlike amplitude and frequency modulation, is used in the often in the optical range [6] than in the radio diapause. Its practical value is found microscopy [7], emitter condition monitoring, diversity reception, interference suppression, and more [8, 9]. This type of modulation gives good indicators of noise immunity, and the speed of information transmission. Polarization modulation is carried out by turning the polarization plane [10] or a change in the type of polarization [11], such modulation is called polarization optical.

In this article, the study of the acceleration of the charged particle will continue with the ultra-short polarization-modulated (PM) laser impulse of high intensity 10^{19} W/cm^2 [12]. The work will reveal the influence of the external magnetic field on the energy characteristics of the particle moving in the PM electromagnetic wave field. The results will be of interest, since the influence of the external physical field on the oscillating particle in the wave field is an important task for the development of real technical systems, such as multi-frequency lasers and ultra-suffering modulation equipment.

The work does not take into account the influence of radiation friction. From the article [13] it is known that the loss of electron energy due to rigid radiation is achieved with energy of 1 GeV, which corresponds to the intensity of the laser field ~10^{22} W/cm^2. In this work, the calculations of all characteristics were carried out with intensity 10^{19} W/cm^2. However, with prolonged interaction of a wave with a particle, even a small parameter of radiation friction can give a significant contribution to the dynamics of the particle, therefore it is assumed that the PM electromagnetic wave in the operation is presented as an ultra-short laser pulse.

Statement of the problem. The paper [12] presents the formulation of the problem of the following form. Assuming that the amplitude of the electromagnetic wave is modulated according to the harmonic law $b(\xi) = b_{\perp 0}\left\{1+\delta_{AM}\cos\left[\omega_0\xi+\delta_{PM}\sin\left(\sigma\omega_0'\xi+\psi_0\right)+\zeta_0\right]\right\}$, and we apply the Jacobi — Anger expansion [14], then we obtain components of electromagnetic wave vectors:

$$E_x = H_y = b_x\left(1+\delta_{AM}\sum_{l=-\infty}^{\infty} J_l\left(\delta_{PM}\right)\cos\hat{\Phi}_l\right)\sum_{n=-\infty}^{\infty} J_n\left(\delta_{FM}\right)\cos\bar{\Phi}_n,$$

$$E_y = -H_x = fb_y\left(1+\delta_{AM}\sum_{l=-\infty}^{\infty} J_l\left(\delta_{PM}\right)\cos\hat{\Phi}_l\right)\sum_{n=-\infty}^{\infty} J_n\left(\delta_{FM}\right)\sin\bar{\Phi}_n, \quad (1)$$

$$E_z = H_z = 0.$$

The z-axis is the direction of propagation of the electromagnetic wave, when the x and y axes coincide with the direction of the semiaxes of the wave polarization ellipse b_x и b_y, moreover $b_x \geq b_y \geq 0$; the relativistic case corresponds to the condition $\xi = t - (z/c)$; ω is carrier frequency of the wave; $f = \pm 1$ is polarization parameter: the upper and lower signs in the expressions for E_y correspond to right and left polarization; δ_{AM} is amplitude modulation depth, $\delta_{AM} \in [0,1]$; ω_0 is modulation frequency; δ_{PM} is polarization modulation depth; $\sigma = \omega_l / \omega_0'$ is coefficient modulation, $\sigma \in [0,1]$, δ_{FM} is frequency modulation depth; ψ_0 is initial phase of the wave, $\psi_0 \in [0, 2\pi]$; $J_n(\delta_{FM})$, $J_l(\delta_{PM})$ are n-th order and l-th order Bessel function; $\bar{\Phi}_n = (\omega + n\omega')\xi + \alpha + n\varphi_0$; $\hat{\Phi}_l = (\omega + l\omega_0')\xi + \alpha + l\varphi_0$.

Motion of a charged particle in the field of a polarization-modulated electromagnetic wave. The equation of motion of a particle of mass m and charge q has the form:

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{1}{c}\left[\mathbf{v} \times \mathbf{H}_\Sigma\right]\right), \tag{2}$$

where $\mathbf{H}_\Sigma = \mathbf{H} + \mathbf{H}_0$, $\mathbf{H}_0 = \mathbf{k}H_0$, $\mathbf{H}$ is electromagnetic wave magnetic field tension vector, $\mathbf{H}_0$ is external magnetic field tension vector.

The solution of Eq. (2) applying system Eq. (1) gives particle momentum components:

$$p_x = \frac{qb_x}{\omega}\left[\sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1 + n\hat{\alpha}} \sin \bar{\Phi}_n + \right.$$

$$+ \frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{n\hat{\alpha} - l\hat{\alpha}_0} \sin\left(\bar{\Phi}_n - \hat{\Phi}_l\right) +$$

$$\left. + \frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{2 + n\hat{\alpha} + l\hat{\alpha}_0} \sin\left(\bar{\Phi}_n + \hat{\Phi}_l\right)\right] + \frac{qH_0}{c}(y - y_0) + \chi_x,$$

$$p_y = \mp \frac{qb_y}{\omega}\left[\sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1 + n\hat{\alpha}} \cos \bar{\Phi}_n + \right.$$

$$+ \frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{n\hat{\alpha} - l\hat{\alpha}_0} \cos\left(\bar{\Phi}_n - \hat{\Phi}_l\right) +$$

$$\left. + \frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{2 + n\hat{\alpha} + l\hat{\alpha}_0} \cos\left(\bar{\Phi}_n + \hat{\Phi}_l\right)\right] - \frac{qH_0}{c}(x - x_0) + \chi_y,$$

$$p_z = \gamma g.$$

Here

$$\chi_x = \frac{mv_{0x}}{\sqrt{1-v_0^2/c^2}} - \frac{qb_x}{\omega}\left\{ \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \sin\bar{\Phi}_{0n} + \right.$$

$$+\frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2} \times$$

$$\left. \times \left[(n\hat{\alpha} - l\hat{\alpha}_0) \sin\left(\bar{\Phi}_{0n} + \hat{\Phi}_{0l}\right) + (2+n\hat{\alpha} + l\hat{\alpha}_0) \sin\left(\bar{\Phi}_{0n} - \hat{\Phi}_{0l}\right)\right]\right\};$$

$$\chi_y = \frac{mv_{0y}}{\sqrt{1-v_0^2/c^2}} + f\frac{qb_y}{\omega}\left\{ \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \cos\bar{\Phi}_{0n} + \right.$$

$$+\frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2} \times$$

$$\left. \times \left[(n\hat{\alpha} - l\hat{\alpha}_0) \cos\left(\bar{\Phi}_{0n} + \hat{\Phi}_{0l}\right) + (2+n\hat{\alpha} + l\hat{\alpha}_0) \cos\left(\bar{\Phi}_{0n} - \hat{\Phi}_{0l}\right)\right]\right\};$$

$$\gamma = \frac{mc(1 - v_{0z}/c)}{\sqrt{1-v_0^2/c^2}}.$$

In order to obtain particle coordinates in the form of a clear dependence on time, it is necessary to obtain and solve a system of differential equations of the second order:

$$\ddot{x} + \omega_c^2 x = \frac{qc}{\gamma} \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \left[b_x (1+n\hat{\alpha}) \mp b_y \eta \right] \cos\bar{\Phi}_n +$$

$$+\frac{qc}{\gamma}\frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{n\hat{\alpha} - l\hat{\alpha}_0} \left[b_x (n\hat{\alpha} - l\hat{\alpha}_0) \mp b_y \eta \right] \cos\left(\bar{\Phi}_n - \hat{\Phi}_l\right) +$$

$$+\frac{qc}{\gamma}\frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{2+n\hat{\alpha} + l\hat{\alpha}_0} \left[b_x (2+n\hat{\alpha} + l\hat{\alpha}_0) \mp b_y \eta \right] \cos\left(\bar{\Phi}_n + \hat{\Phi}_l\right) +$$

$$+\frac{c\omega_c}{\gamma}\left(\chi_y + \frac{\gamma\omega_c}{c} x_0 \right),$$

$$\ddot{y} + \omega_c^2 y = -\frac{qc}{\gamma} \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \left[b_x \eta \mp b_y (1+n\hat{\alpha}) \right] \sin\bar{\Phi}_n -$$

$$-\frac{qc}{\gamma}\frac{\delta_{AM}}{2} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n(\delta_{FM}) J_l(\delta_{PM})}{n\hat{\alpha} - l\hat{\alpha}_0} \left[b_x \eta \mp b_y (n\hat{\alpha} - l\hat{\alpha}_0) \right] \sin\left(\bar{\Phi}_n - \hat{\Phi}_l\right) -$$

$$-\frac{qc}{\gamma}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\left[b_x\eta\mp b_y(2+n\hat{\alpha}+l\hat{\alpha}_0)\right]\times$$

$$\times\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)-\frac{c\omega_c}{\gamma}\left(\chi_x+\frac{\gamma\omega_c}{c}y_0\right), \quad (3)$$

where $\omega_c = qH_0/\gamma$ is cyclotron frequency; $\eta = \omega_c/\omega$.

The solution of the system (3) is the sum of the solution of a homogeneous equation and the partial solution of the heterogeneous equation, considering the initial conditions:

$$x = R\cos\Phi_c - \frac{q}{\gamma\omega k}\sum_{n=-N}^{N}\frac{J_n(\delta_{FM})}{1+n\hat{\alpha}}\frac{\left[b_x(1+n\hat{\alpha})\mp b_y\eta\right]}{(1+n\hat{\alpha})^2-\eta^2}\cos\bar{\Phi}_n -$$

$$-\frac{q}{\gamma\omega k}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{n\hat{\alpha}-l\hat{\alpha}_0}\frac{\left[b_x(n\hat{\alpha}-l\hat{\alpha}_0)\mp b_y\eta\right]}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}\times$$

$$\times\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)-\frac{q}{\gamma\omega k}\frac{\delta_{AM}}{2}\times$$

$$\times\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\frac{\left[b_x(2+n\hat{\alpha}+l\hat{\alpha}_0)\mp b_y\eta\right]}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}\times$$

$$\times\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)+\frac{c}{\gamma\omega_c}\chi_y+x_0,$$

$$y = R\sin\Phi_c + \frac{q}{\gamma\omega k}\sum_{n=-N}^{N}\frac{J_n(\delta_{FM})}{1+n\hat{\alpha}}\frac{\left[b_x\eta\mp b_y(1+n\hat{\alpha})\right]}{(1+n\hat{\alpha})^2-\eta^2}\sin\bar{\Phi}_n +$$

$$+\frac{q}{\gamma\omega k}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{n\hat{\alpha}-l\hat{\alpha}_0}\frac{\left[b_x\eta\mp b_y(n\hat{\alpha}-l\hat{\alpha}_0)\right]}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}\times$$

$$\times\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\frac{q}{\gamma\omega k}\frac{\delta_{AM}}{2}\times$$

$$\times\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\frac{\left[b_x\eta\mp b_y(2+n\hat{\alpha}+l\hat{\alpha}_0)\right]}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}\times$$

$$\times\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)-\frac{c}{\gamma\omega_c}\chi_x+y_0, \quad (4)$$

where R is a constant set by the initial conditions [15].

From the system (4) you can get expressions for the particle pulse in the form of a clear dependence on the time:

$$p_x = \frac{q}{\omega}\left\langle \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \left\{ b_x + \frac{\eta\left[b_x\eta \mp b_y(1+n\hat{\alpha})\right]}{(1+n\hat{\alpha})^2-\eta^2} \right\} \sin\bar{\Phi}_n + \right.$$

$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{n\hat{\alpha}-l\hat{\alpha}_0}\left\{ b_x + \frac{\eta\left[b_x\eta \mp b_y(n\hat{\alpha}-l\hat{\alpha}_0)\right]}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}\right\}\times$$

$$\times\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\frac{\delta_{AM}}{2}\times$$

$$\times\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\times$$

$$\left.\times\left\{ b_x + \frac{\eta\left[b_x\eta \mp b_y(2+n\hat{\alpha}+l\hat{\alpha}_0)\right]}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}\right\}\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\right\rangle+\frac{R\omega_c}{c}\gamma\sin\Phi_c,$$

$$p_y = \frac{q}{\omega}\left\langle \sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \left\{ \mp b_y + \frac{\eta\left[b_x(1+n\hat{\alpha}) \mp b_y\eta\right]}{(1+n\hat{\alpha})^2-\eta^2} \right\} \cos\bar{\Phi}_n + \right.$$

$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{n\hat{\alpha}-l\hat{\alpha}_0}\left\{ \mp b_y + \frac{\eta\left[b_x(n\hat{\alpha}-l\hat{\alpha}_0) \mp b_y\eta\right]}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}\right\}\times$$

$$\times\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\times$$

$$\left.\times\left\{ \mp b_y + \frac{\eta\left[b_x(2+n\hat{\alpha}+l\hat{\alpha}_0) \mp b_y\eta\right]}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}\right\}\cos(\bar{\Phi}_n+\hat{\Phi}_l)\right\rangle-\frac{R\omega_c}{c}\gamma\cos\Phi_c.$$

The expression for the longitudinal component of the particle pulse is adjusted to the relativistic factor γ will be:

$$g = h - \frac{q^2\left(b_{\bar{x}}^2 - b_x^2\right)}{4\gamma^2\omega^2}\times$$

$$\times\left\langle \sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\left\{1+2\frac{\eta^2}{(1+n\hat{\alpha})^2-\eta^2}-\frac{\eta^2(1+n\hat{\alpha})^2-\eta^4}{\left[(1+n\hat{\alpha})^2-\eta^2\right]^2}\right\}\cos 2\bar{\Phi}_n + \right.$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\times$$

$$\times\left\{1+2\frac{\eta^2}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}-\frac{\eta^2(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^4}{\left[(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}\times$$

$$\times\cos 2\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left\{1+2\frac{\eta^2}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}-\frac{\eta^2\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^4}{\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]^2}\right\}\cos 2\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\Bigg\rangle+$$

$$+\frac{q^2\left(b_x^2+b_y^2\right)}{2\gamma^2\omega^2}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l(\delta_{PM})}{1+n\hat{\alpha}}\times$$

$$\times\left\{\frac{1}{n\hat{\alpha}-l\hat{\alpha}_0}\left[1+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(1+n\hat{\alpha}\right)}{\left(1+n\hat{\alpha}\right)^2-\eta^2}+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(n\hat{\alpha}-l\hat{\alpha}_0\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}+\right.\right.$$

$$\left.+\eta^2\frac{\left[\eta^2+\left(1+n\hat{\alpha}\right)\left(n\hat{\alpha}-l\hat{\alpha}_0\right)\right]\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(1+2n\hat{\alpha}-l\hat{\alpha}_0\right)}{\left[\left(1+n\hat{\alpha}\right)^2-\eta^2\right]\left[\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2\right]}\right]+\frac{1}{2+n\hat{\alpha}+l\hat{\alpha}_0}\times$$

$$\times\left[1+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(1+n\hat{\alpha}\right)}{\left(1+n\hat{\alpha}\right)^2-\eta^2}+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}+\right.$$

$$\left.\left.+\eta^2\frac{\left[\eta^2+\left(1+n\hat{\alpha}\right)\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)\right]\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(3+2n\hat{\alpha}+l\hat{\alpha}_0\right)}{\left[\left(1+n\hat{\alpha}\right)^2-\eta^2\right]\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]}\right]\right\}\cos\hat{\Phi}_l+$$

$$+\frac{q^2\left(b_x^2+b_y^2\right)}{2\gamma^2\omega^2}\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{FM})}{\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left\{1+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(n\hat{\alpha}-l\hat{\alpha}_0\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}+\frac{\eta^2\mp 2\frac{b_xb_y}{b_x^2+b_y^2}\eta\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}+\right.$$

$$\left.+\eta^2\frac{\eta^2+\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2\mp 4b_x^2b_y^2\eta\left(1+n\hat{\alpha}\right)}{\left[\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2\right]\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]}\right\}\cos 2\hat{\Phi}_l-$$

$$
-\frac{q^2\left(b_x^2-b_y^2\right)}{2\gamma^2\omega^2}\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{FM}\right)}{\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2}\times
$$

$$
\times\left\{1+\frac{\eta^2}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}+\frac{\eta^2}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}-\right.
$$

$$
\left.-\eta^2\frac{\left[\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2\right]-\eta^2}{\left[\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2\right]\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]}\right\}\cos 2\bar{\Phi}_n-
$$

$$
-\frac{q^2\left(b_x^2-b_y^2\right)}{2\gamma^2\omega^2}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l\left(\delta_{PM}\right)}{\left(1+n\hat{\alpha}\right)\left(n\hat{\alpha}-l\hat{\alpha}_0\right)}\times
$$

$$
\times\left\{1+\frac{\eta^2}{\left(1+n\hat{\alpha}\right)^2-\eta^2}+\frac{\eta^2}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}-\right.
$$

$$
\left.-\eta^2\frac{\left(1+n\hat{\alpha}\right)\left(n\hat{\alpha}-l\hat{\alpha}_0\right)-\eta^2}{\left[\left(1+n\hat{\alpha}\right)^2-\eta^2\right]\left[\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2\right]}\right\}\cos\left(2\bar{\Phi}_n-\hat{\Phi}_l\right)-
$$

$$
-\frac{q^2\left(b_x^2-b_y^2\right)}{2\gamma^2\omega^2}\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l\left(\delta_{PM}\right)}{\left(1+n\hat{\alpha}\right)\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}\times
$$

$$
\times\left\{1+\frac{\eta^2}{\left(1+n\hat{\alpha}\right)^2-\eta^2}+\frac{\eta^2}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}-\right.
$$

$$
\left.-\eta^2\frac{\left(1+n\hat{\alpha}\right)\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)-\eta^2}{\left[\left(1+n\hat{\alpha}\right)^2-\eta^2\right]\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]}\right\}\cos\left(2\bar{\Phi}_n+\hat{\Phi}_l\right)+
$$

$$
+\frac{R\omega_c}{c}\frac{q}{2\gamma\omega}\left\langle\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})}{1+n\hat{\alpha}}\left\{\left(b_x\pm b_y\right)\left[1-\frac{1}{\left(1+n\hat{\alpha}\right)+\eta}\right]\cos\left(\bar{\Phi}_n-\Phi_c\right)-\right.\right.
$$

$$
\left.-\left(b_x\mp b_y\right)\left[1+\frac{1}{\left(1+n\hat{\alpha}\right)-\eta}\right]\cos\left(\bar{\Phi}_n+\Phi_c\right)\right\}+
$$

$$
+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n\left(\delta_{FM}\right)J_l\left(\delta_{PM}\right)}{n\hat{\alpha}-l\hat{\alpha}_0}\left\{\left(b_x\pm b_y\right)\left[1-\frac{1}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)+\eta}\right]\times\right.
$$

$$
\left.\times\cos\left(\bar{\Phi}_n-\hat{\Phi}_l-\Phi_c\right)-\left(b_x\mp b_y\right)\left[1+\frac{1}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)-\eta}\right]\cos\left(\bar{\Phi}_n-\hat{\Phi}_l+\Phi_c\right)\right\}+
$$

$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\left\{(b_x \pm b_y)\left[1-\frac{1}{(2+n\hat{\alpha}+l\hat{\alpha}_0)+\eta}\right]\times\right.$$

$$\times\cos\left(\bar{\Phi}_n+\hat{\Phi}_l-\Phi_c\right)-(b_x \mp b_y)\times$$

$$\left.\left.\times\left[1+\frac{1}{(2+n\hat{\alpha}+l\hat{\alpha}_0)-\eta}\right]\cos\left(\bar{\Phi}_n+\hat{\Phi}_l+\Phi_c\right)\right\}\right\rangle, \qquad (5)$$

$$h=\frac{m^2c^2}{2\gamma^2}-\frac{1}{2}+\frac{R^2\omega_c^2}{2c^2}+\frac{q^2\left(b_x^2+b_x^2\right)}{4\gamma^2\omega^2}\left\langle\sum_{n=N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\times\right.$$

$$\times\left\{1+2\frac{\eta^2 \mp 2\frac{b_xb_y}{b_x^2+b_x^2}\eta(1+n\hat{\alpha})}{(1+n\hat{\alpha})^2-\eta^2}+\eta^2\frac{\left[(1+n\hat{\alpha})^2+\eta^2\right]\mp 4\frac{b_xb_y}{b_x^2+b_x^2}\eta(1+n\hat{\alpha})}{\left[(1+n\hat{\alpha})^2-\eta^2\right]^2}\right\}+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\left\{1+2\frac{\eta^2 \mp 2\frac{b_xb_y}{b_x^2+b_x^2}\eta(n\hat{\alpha}-l\hat{\alpha}_0)}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}+\right.$$

$$\left.+\eta^2\frac{\left((n\hat{\alpha}-l\hat{\alpha}_0)^2+\eta^2\right)\mp 4\frac{b_xb_y}{b_x^2+b_x^2}\eta(n\hat{\alpha}-l\hat{\alpha}_0)}{\left((n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2\right)^2}\right\}+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\left\{1+2\frac{\eta^2 \mp 2\frac{b_xb_y}{b_x^2+b_x^2}\eta(2+n\hat{\alpha}+l\hat{\alpha}_0)}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}+\right.$$

$$\left.\left.+\eta^2\frac{\left[(2+n\hat{\alpha}+l\hat{\alpha}_0)^2+\eta^2\right]\mp 4\frac{b_xb_y}{b_x^2+b_x^2}\eta(2+n\hat{\alpha}+l\hat{\alpha}_0)}{\left[(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}\right\rangle.$$

The expression for the z coordinates in the form of dependence on time can be obtained by interpreting the Eq. (5) according to the spatio-temporal component ξ.

Influence of a constant magnetic field on the energy characteristics of a charged particle. The expression of the average kinetic energy of the charged particle moving in the field of the PM wave and in a constant homogeneous magnetic field will look as follows:

$$
\begin{aligned}
\overline{\varepsilon} = \frac{\gamma c}{1+h}\Bigg\langle & (1+h)^2 + \frac{q^4\left(b_x^2-b_y^2\right)^2}{32\gamma^4\omega^4}\Bigg\{\sum_{n=-N}^{N}\frac{J_n^4(\delta_{FM})}{(1+n\hat{\alpha})^4}\Bigg\{1+2\frac{\eta^2}{(1+n\hat{\alpha})^2-\eta^2}- \\
& -\eta^2\frac{(1+n\hat{\alpha})^2-\eta^2}{\left[(1+n\hat{\alpha})^2-\eta^2\right]^2}\Bigg\}^2 + \frac{\delta_{AM}^4}{16}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^4(\delta_{FM})\,J_l^4(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^4}\times \\
& \times\Bigg\{1+2\frac{\eta^2}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}-\eta^2\frac{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}{\left[(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2\right]^2}\Bigg\}^2+ \\
& +\frac{\delta_{AM}^4}{16}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^4(\delta_{FM})\,J_l^4(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^4}\times \\
& \times\Bigg\{1+2\frac{\eta^2}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}-\eta^2\frac{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}{\left[(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2\right]^2}\Bigg\}^2\Bigg\}+ \\
& +\frac{R^2\omega_c^2}{c^2}\frac{q^2}{8\gamma^2\omega^2}\Bigg\langle\sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\Bigg\{\left(b_x\pm b_y\right)^2\left[1-\frac{1}{(1+n\hat{\alpha})+\eta}\right]^2+ \\
& +\left(b_x\mp b_y\right)^2\left[1+\frac{1}{(1+n\hat{\alpha})-\eta}\right]^2\Bigg\}+ \\
& +\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})\,J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\times \\
& \times\Bigg\{\left(b_x\pm b_y\right)^2\left[1-\frac{1}{(n\hat{\alpha}-l\hat{\alpha}_0)+\eta}\right]^2+\left(b_x\mp b_y\right)^2\left[1+\frac{1}{(n\hat{\alpha}-l\hat{\alpha}_0)-\eta}\right]^2\Bigg\}+ \\
& +\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})\,J_l^2(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\times \\
& \times\Bigg\{\left(b_x\pm b_y\right)^2\left[1-\frac{1}{(2+n\hat{\alpha}+l\hat{\alpha}_0)+\eta}\right]^2+ \\
& +\left(b_x\mp b_y\right)^2\left[1+\frac{1}{(2+n\hat{\alpha}+l\hat{\alpha}_0)-\eta}\right]^2\Bigg\}\Bigg\rangle\Bigg\rangle. \qquad (6)
\end{aligned}
$$

We write down the Eq. (6) in cases of circular and linear polarization of a modulated wave, provided that the initial speed of the particle is not absent.

Circular polarization. In this case, the semiaxes of the wave polarization ellipse are equal ($b_x = b_y = b / \sqrt{2}$):

$$\Psi = mc^2 \frac{\mu}{2} \left\langle \theta + \frac{\mu}{4(1+(\mu/2)\theta)} \left\{ \sum_{n=-N}^{N} \frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2} \left[1 - \frac{f\eta}{(1+n\hat{\alpha})+f\eta} \right]^2 + \right.\right.$$
$$+ \frac{\delta_{AM}^2}{4} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n^2(\delta_{FM}) J_l^2(\delta_{PM})}{(n\hat{\alpha} - l\hat{\alpha}_0)^2} \left[1 - \frac{f\eta}{(n\hat{\alpha} - l\hat{\alpha}_0) + f\eta} \right]^2 +$$
$$\left. + \frac{\delta_{AM}^2}{4} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n^2(\delta_{FM}) J_l^2(\delta_{PM})}{(2+n\hat{\alpha} + l\hat{\alpha}_0)^2} \left[1 - \frac{f\eta}{(2+n\hat{\alpha} + l\hat{\alpha}_0) + f\eta} \right]^2 \right\} \times$$
$$\times \left\{ \sum_{n=-N}^{N} \frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2} \left[1 - \frac{f}{(1+n\hat{\alpha})+f\eta} \right]^2 + \right.$$
$$+ \frac{\delta_{AM}^2}{4} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n^2(\delta_{FM}) J_l^2(\delta_{PM})}{(n\hat{\alpha} - l\hat{\alpha}_0)^2} \times$$
$$\times \left[1 - \frac{f\eta}{(n\hat{\alpha} - l\hat{\alpha}_0) + f\eta} \right]^2 + \frac{\delta_{AM}^2}{4} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n^2(\delta_{FM}) J_l^2(\delta_{PM})}{(2+n\hat{\alpha} + l\hat{\alpha}_0)^2} \times$$
$$\left.\left. \times \left[1 - \frac{f\eta}{(2+n\hat{\alpha} + l\hat{\alpha}_0) + f\eta} \right]^2 \right\} \right\rangle.$$

Here $\mu = q^2 b^2 / (\gamma^2 \omega^2) = I \lambda^2 2q^2 / (\pi m^2 c^5)$;

$$\theta = \sum_{n=-N}^{N} \frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2} \left\{ \frac{1 + \left[\dfrac{f\eta}{(1+n\hat{\alpha})+f\eta} \right]^3}{1 + \dfrac{f\eta}{(1+n\hat{\alpha})+f\eta}} - \frac{f\eta}{(1+n\hat{\alpha})+f\eta} \right\} +$$
$$+ \frac{\delta_{AM}^2}{4} \sum_{n=-N}^{N} \sum_{l=-L}^{L} \frac{J_n^2(\delta_{FM}) J_l^2(\delta_{PM})}{(n\hat{\alpha} - l\hat{\alpha}_0)^2} \times$$
$$\times \left\{ \frac{1 + \left[\dfrac{f\eta}{(n\hat{\alpha} - l\hat{\alpha}_0)+f\eta} \right]^3}{1 + \dfrac{f\eta}{(n\hat{\alpha} - l\hat{\alpha}_0)+f\eta}} - \frac{f\eta}{(n\hat{\alpha} - l\hat{\alpha}_0)+f\eta} \right\} +$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\times$$

$$\times\left\{\frac{1+\left[\dfrac{f\eta}{(2+n\hat{\alpha}+l\hat{\alpha}_0)+\eta}\right]^3}{1+\dfrac{f\eta}{(2+n\hat{\alpha}+l\hat{\alpha}_0)+f\eta}}-\frac{f\eta}{(2+n\hat{\alpha}+l\hat{\alpha}_0)+f\eta}\right\};$$

$\Psi=\overline{\varepsilon}-mc^2$ is the energy of a particle without considering its rest energy.

Linear polarization. In this case, one of the semiaxis of the wave polarization ellipse is zero, and the other is equal to b $(b_x=b; b_y=0)$:

$$\Psi=mc^2\frac{\mu}{4}\left\langle 2\sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\left\{1+\frac{2\eta^2}{(1+n\hat{\alpha})^2-\eta^2}+\frac{\eta^2(1+n\hat{\alpha})^2+\eta^4}{\left[(1+n\hat{\alpha})^2-\eta^2\right]^2}\right\}+\right.$$

$$+\frac{\delta_{AM}^2}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\times$$

$$\times\left\{1+\frac{2\eta^2}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}+\frac{\eta^2(n\hat{\alpha}-l\hat{\alpha}_0)^2+\eta^4}{\left[(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}+$$

$$+\frac{\delta_{AM}^2}{2}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\times$$

$$\times\left\{1+\frac{2\eta^2}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}+\frac{\eta^2(2+n\hat{\alpha}+l\hat{\alpha}_0)^2+\eta^4}{\left[(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}+$$

$$+2\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\left[\left(1-\frac{1}{1+n\hat{\alpha}+\eta}\right)^2+\left(1+\frac{1}{1+n\hat{\alpha}-\eta}\right)^2\right]+\right.$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\times$$

$$\times\left[\left(1-\frac{1}{n\hat{\alpha}-l\hat{\alpha}_0+\eta}\right)^2+\left(1+\frac{1}{n\hat{\alpha}-l\hat{\alpha}_0-\eta}\right)^2\right]+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\times$$

$$\times\left[\left(1-\frac{1}{2+n\hat{\alpha}+l\hat{\alpha}_0+\eta}\right)^2+\left(1+\frac{1}{2+n\hat{\alpha}+l\hat{\alpha}_0-\eta}\right)^2\right]\Bigg\}+$$

$$+\frac{\mu}{2\sqrt{\mu(2\beta-\kappa)+4}\sqrt{\mu(2\beta+\kappa)+4}}\times$$

$$\times\left\{\sum_{n=-N}^{N}\frac{J_n^4(\delta_{FM})}{(1+n\hat{\alpha})^4}\left\{1+2\frac{\eta^2}{(1+n\hat{\alpha})^2-\eta^2}-\frac{\eta^2(1+n\hat{\alpha})^2-\eta^4}{\left[(1+n\hat{\alpha})^2-\eta^2\right]^2}\right\}^2+\right.$$

$$+\frac{\delta_{AM}^4}{16}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^4(\delta_{FM})J_l^4(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^4}\times$$

$$\times\left\{1+2\frac{\eta^2}{(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2}-\frac{\eta^2(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^4}{\left[(n\hat{\alpha}-l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}^2+$$

$$+\frac{\delta_{AM}^4}{16}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^4(\delta_{FM})J_l^4(\delta_{PM})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^4}\times$$

$$\left.\times\left\{1+2\frac{\eta^2}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2}-\frac{\eta^2(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^4}{\left[(2+n\hat{\alpha}+l\hat{\alpha}_0)^2-\eta^2\right]^2}\right\}^2\right\}-$$

$$-\frac{4\mu\beta+16}{2\sqrt{\mu(2\beta-\kappa)+4}\sqrt{\mu(2\beta+\kappa)+4}}\times$$

$$\times\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}\left[\left(1-\frac{1}{1+n\hat{\alpha}+\eta}\right)^2+\left(1+\frac{1}{1+n\hat{\alpha}-\eta}\right)^2\right]+\right.$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2(\delta_{FM})J_l^2(\delta_{PM})}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\times$$

$$\times\left[\left(1-\frac{1}{n\hat{\alpha}-l\hat{\alpha}_0+\eta}\right)^2+\left(1+\frac{1}{n\hat{\alpha}-l\hat{\alpha}_0-\eta}\right)^2\right]+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{PM}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left[\left(1-\frac{1}{2+n\hat{\alpha}+l\hat{\alpha}_0+\eta}\right)^2+\left(1+\frac{1}{2+n\hat{\alpha}+l\hat{\alpha}_0-\eta}\right)^2\right]\Bigg\}\Bigg\rangle,$$

where

$$\kappa=\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{FM}\right)}{\left(1+n\hat{\alpha}\right)^2}\left[1+\frac{2\eta^2}{\left(1+n\hat{\alpha}\right)^2-\eta^2}\right]+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{PM}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\times$$

$$\times\left[1+\frac{2\eta^2}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}\right]+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{PM}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left[1+\frac{2\eta^2}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}\right];$$

$$\beta=\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{FM}\right)}{\left(1+n\hat{\alpha}\right)^2}\left\{1+\frac{2\eta^2}{\left(1+n\hat{\alpha}\right)^2-\eta^2}+\frac{\eta^2\left(1+n\hat{\alpha}\right)^2+\eta^4}{\left[\left(1+n\hat{\alpha}\right)^2-\eta^2\right]^2}\right\}+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{PM}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\times$$

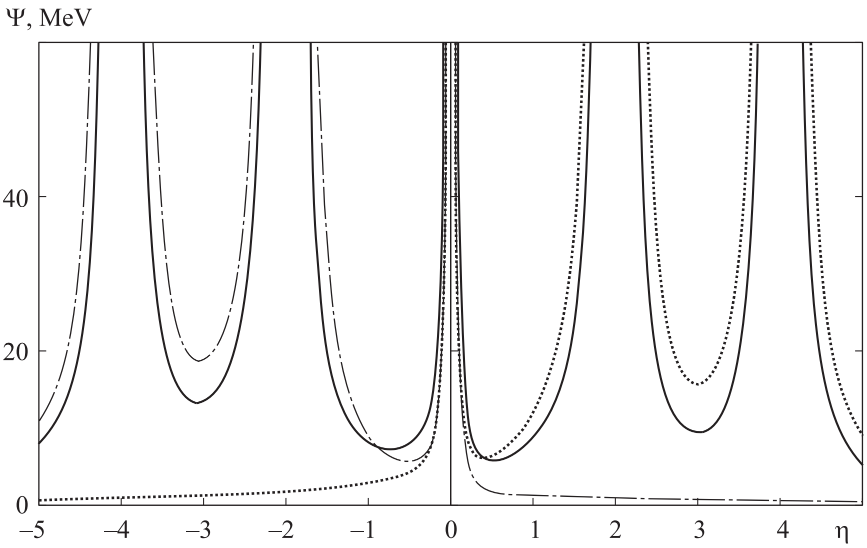
$$\times\left\{1+\frac{2\eta^2}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2}+\frac{\eta^2\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2+\eta^4}{\left[\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2-\eta^2\right]^2}\right\}+$$

$$+\frac{\delta_{AM}^2}{4}\sum_{n=-N}^{N}\sum_{l=-L}^{L}\frac{J_n^2\left(\delta_{FM}\right)J_l^2\left(\delta_{PM}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left\{1+\frac{2\eta^2}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2}+\frac{\eta^2\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2+\eta^4}{\left[\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2-\eta^2\right]^2}\right\}.$$

Now it is possible to display the influence of a constant magnetic field on the energy characteristics of a charged particle moving in the field of polarization-modulated electromagnetic wave.

Figure shows that in the case of a modulated wave polarization, there are five sections of autoresonance instead of three sections, as is observed in the case of amplitude or frequency modulation [14].



The influence of the magnetic field on the energy characteristics of the particle in the PM electromagnetic wave of the linear (——) and circular (right (–·–) and left (·······)) polarization

Conclusion. This paper presents an analytical solution of the equation of motion of a charged particle in the field of a PM electromagnetic wave and an external permanent magnetic field. The energy characteristics of a charged particle moving in this configuration of fields are determined and it is shown on the graph of the dependence of the average kinetic energy of the particle on the magnitude of the external physical field that in the case of modulation of the polarization of an electromagnetic wave and the presence of an external homogeneous magnetic field, five sections of autoresonance are observed instead of three, as occurs in cases of frequency or amplitude modulation of the wave. Note that in the absence of a magnetic field $(\eta = 0)$, the formulas for the energy characteristics of the particle change to the form presented in [11].

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