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ON ONE APPROACH TO THE SOLUTION OF THE LAMBERT PROBLEM USING THE DECOMPOSITIONAL METHOD OF MODAL CONTROL

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Abstract

A new approach to the solution of the Lambert's problem in spaceflight mechanics is proposed for elliptical orbits. The system of four transcendental algebraic equations is solved using the method of modal synthesis which is based on multilevel decomposition of discrete dynamic system and applied to solve the problem of identification of parameters of discrete system by a state observer. The solution algorithm is as follows: conditional and identification discrete models (systems) are built for the specified system of equations; initial values of estimates are given; initial conditions in the equations of residuals are formed. Using the method of modal synthesis, the problem of search for control of the auxiliary system is solved, as a result of which the matrix of state observer feedback coefficients is calculated. This matrix is used to predict the state vector and to obtain refined estimates — parameters of the planar orbit. A numerical example of the Lambert's problem solution using the proposed algorithm is given. In essence, an approach to the solution of nonlinear algebraic systems of the fourth order, which can be extended to systems of any observable order, is proposed. The peculiarity of the proposed algorithm is that the convergence of the iterative process of finding a solution can have a different "adjustable" speed using the control law

Keywords

Lambert's problem, elliptic orbits, discrete modelling, state observer, modal synthesis

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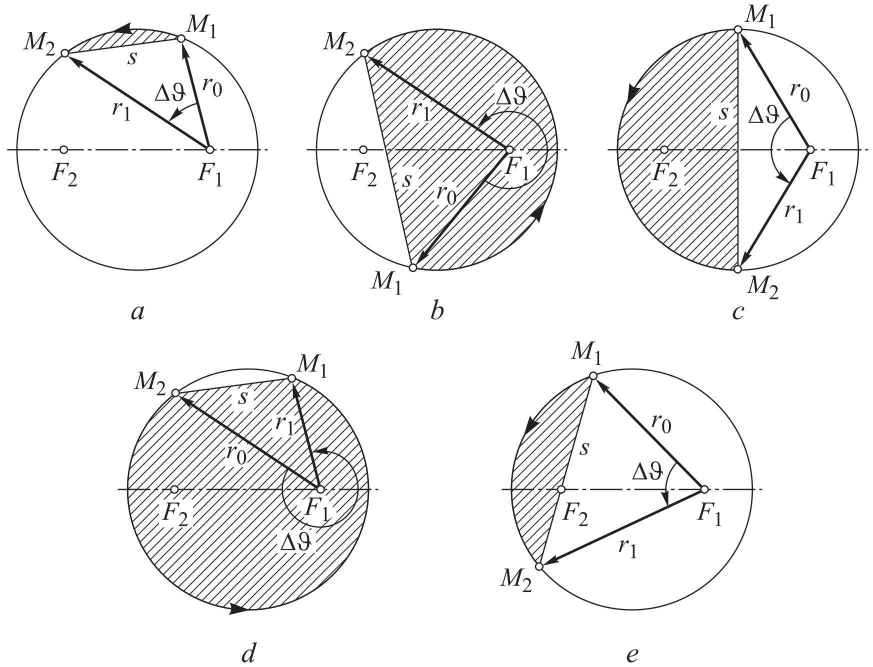
Introduction. The Lambert's problem is one of the important problems of spaceflight mechanics. Despite numerous and exhaustive investigations in this direction, the Lambert's problem still remains very attractive for researchers [1–9].

We restrict ourselves to the case of elliptical orbits and consider it as a problem of determining elements of a Kepler orbit within one revolution by two positions of the spacecraft (SC). In the considered case, the Lambert's problem is formulated as follows: from the two positions of the SC in the orbit in the form of measurements of the moduli of the radius vectors (r_0, r_1), of time Δt, elapsed during the flight from one position to another, the difference of the true anomalies $\Delta\vartheta$ is required to determine the semi-major axis *a*, eccentricity *e* and eccentric anomalies E_0, E_1. Accordingly, for elliptic orbits there is a system of four transcendental algebraic equations [10, 11]:

$$\begin{aligned}
&r_0 = a\left(1 - e\cos E_0\right), \\
&r_1 = a\left(1 - e\cos E_1\right), \\
&\Delta\vartheta = 2\,\mathrm{arctg}\left(\sqrt{\frac{1+e}{1-e}}\,\mathrm{tg}\,\frac{E_1}{2}\right) - 2\,\mathrm{arctg}\left(\sqrt{\frac{1+e}{1-e}}\,\mathrm{tg}\,\frac{E_0}{2}\right), \\
&\Delta t = \frac{a^{3/2}}{\sqrt{\mu}}\left(E_1 - E_0 - e\left(\sin E_1 - \sin E_0\right)\right),
\end{aligned} \tag{1}$$

which must be resolved with respect to the four orbital parameters (a, e, E_0, E_1). In (1) μ — gravitational parameter, for the Earth μ = 398600.4418 km^3/s^2. Depending on the variants of the two measurements on the orbit, the possible variants of the classification of trajectories of motion of the solution of the Lambert's problem within one turn are presented in figure [11]. When considering system (1), it should be noted that due to the peculiarities of the arctangent associated with the domain of its values, the application of additional measures is required. For single rotation determination of elliptical orbits, one of the measures is the condition of limited time elapsed between the two positions of the object for which the measurement parameters were determined. In this case, the algorithm can only use the variant shown in figure *a*. It is also necessary to satisfy the rank criterion, the essence of which will be described below. These two circumstances allow us to exclude non-existence and ambiguity of the system solution (1).

In general, all methods for solving this problem are reduced to obtaining and solving a single Lambert equation or a system of two Kepler equations based on the energy integral and the area integral for unperturbed motion.



Classification of elliptic trajectories of flight in the Lambert's problem:
a — sector *s* does not contains F_1 and F_2 focuses; *b* — sector *s* contains F_1 and F_2 focuses; *c* — sector *s* does not contain F_2 focus and contains F_1 focus; *d* — sector *s* does not contain F_1 focus and contains F_2 focus; *e* — sector *s* does not contain F_1 focus, F_2 focus is on the border of this sector

In this paper, we consider an approach based on solving the above system of four equations using the method of modal synthesis. The method is based on a multilevel decomposition of a discrete dynamical system and is applied to solve the problem of identifying the parameters of a discrete system using a state observer. The essence of the solution search is as follows.

Algorithm of the problem solution. Let us construct a conditional dynamic discrete system (model) of the following type:

$$x_p^D(\tau+1) = A^D x_p^D(\tau),\ y(\tau) = C_p x_p^D(\tau), \tag{2}$$

where $x_p^D = \begin{pmatrix} r_0 & r_1 & \Delta\vartheta & \Delta t & a & e & E_0 & E_1 \end{pmatrix}^T$ is a state vector, $\tau = 0, 1, 2, \ldots, N$ is a discrete time determined on the basis of the introduction of a uniform grid of *N* intervals with a step on the segment of the search time of integration constants $h = \Delta\tau / N$. In fact, *N* determines the number of iterations to be performed to solve the problem with a given accuracy. The corresponding matrices in equations (2) have the form $A^D = I_{8\times 8}, C_p = \begin{pmatrix} I_{4\times 4} & 0_{4\times 4} \end{pmatrix}$, where $I_{n\times n}$ is the unit matrix of size $n \times m$.

From (2) it follows that for the subvector $x^D = x = \begin{pmatrix} r_0 & r_1 & \Delta\vartheta & \Delta t \end{pmatrix}^T$ the iteration $x^D(\tau+1) = x^D(\tau)$ is valid.

For the other subvector we denote $x_\tau^D = x_\tau = \begin{pmatrix} a & e & E_0 & E_1 \end{pmatrix}^T$ and present the system (1) in vector form $x(\tau) = G(x_\tau)$:

$$G(x_\tau)=\begin{pmatrix} a\left(1-e\cos E_0\right) \\ a\left(1-e\cos E_1\right) \\ 2\operatorname{arctg}\left(\sqrt{\dfrac{1+e}{1-e}}\right)\operatorname{tg}\dfrac{E_1}{2}-2\operatorname{arctg}\left(\sqrt{\dfrac{1+e}{1-e}}\right)\operatorname{tg}\dfrac{E_0}{2} \\ \dfrac{a^{3/2}}{\sqrt{\mu}}\left(E_1-E_0-e\left(\sin E_1-\sin E_0\right)\right) \end{pmatrix}.$$

For system (2) let us construct a full rank observer of the state, which in general form is defined by the equation [8]:

$$\hat{x}_p^D(\tau+1) = \left(A^D - L_p C_p\right)\hat{x}_p^D(\tau) + L_p y(\tau). \tag{3}$$

Here L_p is the matrix of state observer feedback coefficients [8].

The estimate is written as

$$\hat{x}(\tau) = \hat{G}(\hat{x}_\tau). \tag{4}$$

We linearize the function $G(x_\tau)$ in the vicinity of $\hat{x}_\tau$, using the Taylor series expansion:

$$G(x_\tau) = G(\hat{x}_\tau) + \frac{\partial G(\hat{x}_\tau)}{\partial x_\tau}\tilde{x}_\tau,$$

where $\tilde{x}_\tau = x_\tau - \hat{x}_\tau$; $\partial G(\hat{x}_\tau)/\partial x_\tau$ is the Jacobian matrix. By calculating the discrepancy vector of the $\tilde{x}$ $(\tilde{x} = x - \hat{x})$, we obtain

$$\tilde{x}(\tau+1) = \frac{\partial G(\hat{x}_\tau, t)}{\partial x_\tau}\tilde{x}_\tau(\tau). \tag{5}$$

Combining $\tilde{x}$ and $\tilde{x}_\tau$ into a single vector and using (5) taking into account (3) we obtain the equations of the discrete model of discrepancies:

$$\tilde{x}_p(\tau+1) = \left(A_p^D - L_p C_p\right)\tilde{x}_p(\tau), \tag{6}$$

$$A_p^D = \begin{pmatrix} I_{4\times4} & \dfrac{\partial G(\hat{x}_\tau, t)}{\partial x_\tau} \\ 0 & I_{4\times4} \end{pmatrix}.$$

If the condition of full observability is fulfilled

$$\operatorname{rank}\begin{pmatrix} C_p \\ C_p A_p^D \\ C_p\left(A_p^D\right)^2 \\ C_p\left(A_p^D\right)^3 \\ C_p\left(A_p^D\right)^4 \end{pmatrix} = 4+4=8, \tag{7}$$

the choice of the coefficient matrix L_p with known matrices A_p^D and C_p it is always possible to provide any given placement on the complex plane $\mathbb{C}^{\text{stab}}$ (in the considered example, the area inside a circle of unit radius) of a characteristic polynomial root (poles) [12] $\det\left(\lambda I_n - \left(A_p^D - L_p C_p\right)\right)$ or equivalent to the set of eigenvalues

$$\operatorname{eig}\left(A_p^D - L_p C_p\right) = \left\{\lambda_i \in \mathbb{C} : \det\left(\lambda I_{n+m} - \left(A_p^D - L_p C_p\right)\right) = 0\right\}$$

state observer and thereby solve the equations system (1). In this case it is necessary to consider an auxiliary discrete model of the form

$$\nu(\tau+1) = D^T \nu(\tau) + B^T \eta(\tau), \quad \eta(\tau) = -L_p^T \nu(\tau), \tag{8}$$

where ν is a vector having the dimensionality of the extended vector x_p; η is the control vector; $D^T = \left(A_p^D\right)^T$; $C^T = \left(C_p\right)^T$.

The search for the L_p matrix belongs to the classical problem of modal control and in the case under consideration is in fact the goal in determining a, e, E_0, E_1, since their identification is the solution of the problem posed earlier.

If the condition of complete observability is fulfilled, the matrices necessary for the solution of the control problem exist. To solve the observer synthesis problem, any of the modal control methods can be applied. Here, as in [12], we use the decomposition method [12–15].

Let us introduce a multilevel decomposition of the discrete system (8) represented by a pair of matrices $\left(D^T,\ B^T\right)$. We have zero (initial) level

$$A_0 = D^T, B_0 = C^T, \tag{9}$$

the first level

$$A_1 = B_0^{\perp} A_0 B_0^{\perp -}, \ B_1 = B_0^{\perp} A_0 B_0. \tag{10}$$

Here $B_0^{\perp}$ is the annulator (divisor of zero) of the matrix B_0, i.e., $B_0^{\perp} B_0 = 0$; $B_0^{\perp -}$ is a two-semi-inverse (2-semi-inverse) matrix for $B_0^{\perp}$ [12], i.e., a matrix satisfying the regularity conditions

$$B_0^{\perp} B_0^{\perp -} B_0^{\perp} = B_0^{\perp}, \; B_0^{\perp -} B_0^{\perp} B_0^{\perp -} = B_0^{\perp -}. \tag{11}$$

Then, according to [8], the required matrix $L = L_0 \in \mathbb{R}^{2\times 4}$ is calculated by the recursive formulas

$$L_1 = B_1^{+} A_1 - \Phi_1 B_1^{+}, \tag{12}$$

$$L_0 = B_0^{-} A_0 - \Phi_0 B_0^{-}, B_0^{-} = L_1 B_0^{\perp} + B_0^{+} \tag{13}$$

and provides an exact given placement of the poles. This is indeed true since all elements of the set of eigenvalues $\operatorname{eig}\left(A - LB\right)$ coincide with the eigenvalues of the given stable matrices $\Phi_k \; (k = 0, 1),$ i.e., eigenvalues lying inside the unit circle.

Considering the above, we write down the algorithm for solving the problem of finding the parameters $a,\; e, E_0,\; E_1$.

1. Using the system of equations (1) models are constructed: conditional (2) and identification (9).

2. Initial values of $\hat{x}_\tau$ estimates are set. It should be noted that an approximate method of calculation proposed in [1, 6] can be used to select the initial values of $\hat{x}_\tau$, which will significantly reduce the number of iterations to find the exact solution.

3. Based on the given initial conditions according to (5), estimates of the state vector x are determined, thus, initial conditions for the difference discrete equation of residuals (6) are formed.

4. Using the method of modal control [12], defined by (9)–(13), we solve the problem of search for control of auxiliary system (8), which results in the transpose matrix L_p^T of observer's feedback coefficients.

5. Using L_p^T values on the basis of (3), we find new estimates of the vector $\hat{x}_\tau$ and then, according to (4), new estimates of the predicted state vector $\hat{x}$.

The matrix $\hat{A}_p^D$ included in the equation of residuals of the form (6):

$$\hat{A}_p^D = \begin{pmatrix} 1 & 0 & 0 & 0 & b_{11} & b_{12} & b_{13} & b_{14} \\ 0 & 1 & 0 & 0 & b_{21} & b_{22} & b_{23} & b_{24} \\ 0 & 0 & 1 & 0 & b_{31} & b_{32} & b_{33} & b_{34} \\ 0 & 0 & 0 & 1 & b_{41} & b_{42} & b_{43} & b_{44} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

where

$$b_{11}=1-e\cos E_0;\ b_{12}=-a\cos E_0;\ b_{13}=ae\sin E_0;\ b_{14}=0;$$

$$b_{21}=1-e\cos E_1;\ b_{22}=-a\cos E_1;\ b_{23}=0;\ b_{24}=ae\sin E_0;$$

$$b_{31}=0;\ b_{32}=\sin E_1/\Big(-(e+1)/\Big((e-1)^{1/2}(e-1)\big(e\cos E_1-1\big)-$$

$$-\sin E_0\Big)\Big)/\Big((-(e+1)/(e-1))^{1/2}(e-1)\big(e\cos E_0\big)-1)\Big);$$

$$b_{33}=-\Big((2(-(e+1)/(e-1))^{1/2}(e-1))/\big(2e\cos E_0-2\big)\Big);$$

$$b_{34}=\Big(2(-(e+1)/(e-1))^{1/2}(e-1)\Big)/\big(2e\cos E_1-2\big);$$

$$b_{41}=\Big(3a^{1/2}\big(E_1-E_0+e\big(\sin E_0-\sin E_1\big)\big)\Big)/\big(2\mu^{1/2}\big);$$

$$b_{42}=a^{3/2}\big(\sin E_0-\sin E_1\big)/\mu^{1/2};$$

$$b_{43}=a^{3/2}\big(e\cos E_0-1\big)/\mu^{1/2};$$

$$b_{44}=-\Big(a^{3/2}\big(e\cos E_1-1\big)\Big)/\mu^{1/2}.$$

Main result. Let us apply the above approach (expressions (8)–(13)) to the problem of identification of elliptical orbit parameters. According to (8) and on the basis of (9), we have

$$A^T=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ b_{11} & b_{21} & b_{31} & b_{41} & 1 & 0 & 0 & 0\\ b_{12} & b_{22} & b_{32} & b_{42} & 0 & 1 & 0 & 0\\ b_{13} & b_{23} & b_{33} & b_{43} & 0 & 0 & 1 & 0\\ b_{14} & b_{24} & b_{34} & b_{44} & 0 & 0 & 0 & 1 \end{pmatrix},\quad C^T=\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

According to (9), the zero level of decomposition for the considered subsystems is $A_0=A^T, B_0=C^T.$ To find the first level of decomposition let us calculate the matrices-annulators:

$$\big(B_0\big)^{\perp}=\begin{pmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix},\quad \big(B_0\big)^{\perp-}=\big(B_0\big)^{\perp T}.$$

Then according to (10), we obtain

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} b_{11} & b_{21} & b_{31} & b_{41} \\ b_{12} & b_{22} & b_{32} & b_{42} \\ b_{13} & b_{23} & b_{33} & b_{43} \\ b_{14} & b_{24} & b_{34} & b_{44} \end{pmatrix}.$$

In order to use expressions (12), (13), let us define a generalized inverse

$$B_0^+ = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and select (assign) the Φ_1, Φ_0 matrices for the corresponding subsystems in the following simple form:

$$\Phi_1^1 = \begin{pmatrix} \lambda_{11} & 0 & 0 & 0 \\ 0 & \lambda_{12} & 0 & 0 \\ 0 & 0 & \lambda_{13} & 0 \\ 0 & 0 & 0 & \lambda_{14} \end{pmatrix}, \quad \Phi_0^1 = \begin{pmatrix} \lambda_{01} & 0 & 0 & 0 \\ 0 & \lambda_{02} & 0 & 0 \\ 0 & 0 & \lambda_{03} & 0 \\ 0 & 0 & 0 & \lambda_{04} \end{pmatrix}.$$

Assume that all the elements are located inside the $\mathbb{C}^{\text{stab}} \doteq \mathbb{C}_{|\lambda|<1}$. Then according to (13), we obtain

$$L_1^T = \begin{pmatrix} (\lambda_{11}-1)l_{11}/D & -(\lambda_{12}-1)l_{12}/D & (\lambda_{13}-1)l_{13}/D & -(\lambda_{14}-1)l_{14}/D \\ -(\lambda_{11}-1)l_{21}/D & (\lambda_{12}-1)l_{22}/D & -(\lambda_{13}-1)l_{23}/D & (\lambda_{14}-1)l_{24}/D \\ (\lambda_{11}-1)l_{31}/D & -(\lambda_{12}-1)l_{32}/D & (\lambda_{13}-1)l_{33}/D & -(\lambda_{14}-1)l_{34}/D \\ -(\lambda_{11}-1)l_{41}/D & (\lambda_{12}-1)l_{42}/D & -(\lambda_{13}-1)l_{43}/D & (\lambda_{14}-1)l_{44}/D \end{pmatrix},$$

where

$$l_{11} = b_{22}b_{33}b_{44} - b_{22}b_{34}b_{43} - b_{23}b_{32}b_{44} + b_{23}b_{34}b_{42} + b_{24}b_{32}b_{43} - b_{24}b_{33}b_{42};$$

$$l_{12} = b_{21}b_{33}b_{44} - b_{21}b_{34}b_{43} - b_{23}b_{31}b_{44} + b_{23}b_{34}b_{41} + b_{24}b_{31}b_{43} - b_{24}b_{33}b_{41};$$

$$l_{13} = b_{21}b_{32}b_{44} - b_{21}b_{34}b_{42} - b_{22}b_{31}b_{44} + b_{22}b_{34}b_{41} + b_{24}b_{31}b_{42} - b_{24}b_{32}b_{41};$$

$$l_{14} = b_{21}b_{32}b_{43} - b_{21}b_{33}b_{42} - b_{22}b_{31}b_{43} + b_{22}b_{33}b_{41} + b_{23}b_{31}b_{42} - b_{23}b_{32}b_{41};$$

$$l_{21} = b_{12}b_{33}b_{44} - b_{12}b_{34}b_{43} - b_{13}b_{32}b_{44} + b_{13}b_{34}b_{42} + b_{14}b_{32}b_{43} - b_{14}b_{33}b_{42};$$

$$l_{22} = b_{11}b_{33}b_{44} - b_{11}b_{34}b_{43} - b_{13}b_{31}b_{44} + b_{13}b_{34}b_{41} + b_{14}b_{31}b_{43} - b_{14}b_{33}b_{41};$$

$$l_{23} = b_{11}b_{32}b_{44} - b_{11}b_{34}b_{42} - b_{12}b_{31}b_{44} + b_{12}b_{34}b_{41} + b_{14}b_{31}b_{42} - b_{14}b_{32}b_{41};$$

$$l_{24} = b_{11}b_{32}b_{43} - b_{11}b_{33}b_{42} - b_{12}b_{31}b_{43} + b_{12}b_{33}b_{41} + b_{13}b_{31}b_{42} - b_{13}b_{32}b_{41};$$

$$l_{31} = b_{12}b_{23}b_{44} - b_{12}b_{24}b_{43} - b_{13}b_{22}b_{44} + b_{13}b_{24}b_{42} + b_{14}b_{22}b_{43} - b_{14}b_{23}b_{42};$$
$$l_{32} = b_{11}b_{23}b_{44} - b_{11}b_{24}b_{43} - b_{13}b_{21}b_{44} + b_{13}b_{24}b_{41} + b_{14}b_{21}b_{43} - b_{14}b_{23}b_{41};$$
$$l_{33} = b_{11}b_{22}b_{44} - b_{11}b_{24}b_{42} - b_{12}b_{21}b_{44} + b_{12}b_{24}b_{41} + b_{14}b_{21}b_{42} - b_{14}b_{22}b_{41};$$
$$l_{34} = b_{11}b_{22}b_{43} - b_{11}b_{23}b_{42} - b_{12}b_{21}b_{43} + b_{12}b_{23}b_{41} + b_{13}b_{21}b_{42} - b_{13}b_{22}b_{41};$$
$$l_{41} = b_{12}b_{23}b_{34} - b_{12}b_{24}b_{33} - b_{13}b_{22}b_{34} + b_{13}b_{24}b_{32} + b_{14}b_{22}b_{33} - b_{14}b_{23}b_{32};$$
$$l_{42} = b_{11}b_{23}b_{34} - b_{11}b_{24}b_{33} - b_{13}b_{21}b_{34} + b_{13}b_{24}b_{31} + b_{14}b_{21}b_{33} - b_{14}b_{23}b_{31};$$
$$l_{43} = b_{11}b_{22}b_{34} - b_{11}b_{24}b_{32} - b_{12}b_{21}b_{34} + b_{12}b_{24}b_{31} + b_{14}b_{21}b_{32} - b_{14}b_{22}b_{31};$$
$$l_{44} = b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31};$$

$$\begin{gathered} D = b_{11}b_{22}b_{33}b_{44} - b_{11}b_{22}b_{34}b_{43} - b_{11}b_{23}b_{32}b_{44} + b_{11}b_{23}b_{34}b_{42} + b_{11}b_{24}b_{32}b_{43} - \\ - b_{11}b_{24}b_{33}b_{42} - b_{12}b_{21}b_{33}b_{44} + b_{12}b_{21}b_{34}b_{43} + b_{12}b_{23}b_{31}b_{44} - b_{12}b_{23}b_{34}b_{41} - \\ - b_{12}b_{24}b_{31}b_{43} + b_{12}b_{24}b_{33}b_{41} + b_{13}b_{21}b_{32}b_{44} - b_{13}b_{21}b_{34}b_{42} - b_{13}b_{22}b_{31}b_{44} + \\ + b_{13}b_{22}b_{34}b_{41} + b_{13}b_{24}b_{31}b_{42} - b_{13}b_{24}b_{32}b_{41} - b_{14}b_{21}b_{32}b_{43} + b_{14}b_{21}b_{33}b_{42} + \\ + b_{14}b_{22}b_{31}b_{43} - b_{14}b_{22}b_{33}b_{41} - b_{14}b_{23}b_{31}b_{42} + b_{14}b_{23}b_{32}b_{41}; \\ D^z = b_{11}^z b_{22}^z - b_{12}^z b_{21}^z. \end{gathered}$$

Using (13), for each subsystem we can calculate the transpose L_p^T matrix of observer's feedbacks. Taking into account its cumbersomeness, we allocate a block concerning the estimation of orbit parameters resulting in

$$L_\tau = \\ = \frac{1}{D}\begin{pmatrix} -(\lambda_{01}-1)(\lambda_{11}-1)l_{11} & (\lambda_{02}-1)(\lambda_{11}-1)l_{21} & -(\lambda_{03}-1)(\lambda_{11}-1)l_{31} & (\lambda_{04}-1)(\lambda_{11}-1)l_{41} \\ (\lambda_{01}-1)(\lambda_{12}-1)l_{12} & -(\lambda_{02}-1)(\lambda_{12}-1)l_{22} & (\lambda_{03}-1)(\lambda_{12}-1)l_{32} & -(\lambda_{04}-1)(\lambda_{12}-1)l_{42} \\ -(\lambda_{01}-1)(\lambda_{13}-1)l_{13} & (\lambda_{02}-1)(\lambda_{13}-1)l_{23} & -(\lambda_{03}-1)(\lambda_{13}-1)l_{33} & (\lambda_{04}-1)(\lambda_{13}-1)l_{43} \\ (\lambda_{01}-1)(\lambda_{14}-1)l_{14} & -(\lambda_{02}-1)(\lambda_{14}-1)l_{24} & (\lambda_{03}-1)(\lambda_{14}-1)l_{34} & -(\lambda_{04}-1)(\lambda_{14}-1)l_{44} \end{pmatrix}.$$

According to (5), the equations of estimates will be written as follows:

$$\begin{pmatrix} \hat{a} \\ \hat{e} \\ \Delta\hat{E}_0 \\ \Delta\hat{E}_1 \end{pmatrix} = L_\tau^T \begin{pmatrix} \tilde{r}_0 \\ \tilde{r}_1 \\ \Delta\tilde{\vartheta} \\ \Delta\tilde{t} \end{pmatrix},$$

accordingly, the values of estimates for the next calculation step with regard to the previous one will be equal to

$$\begin{pmatrix} \hat{a} \\ \hat{e} \\ \Delta\hat{E}_0 \\ \Delta\hat{E}_1 \end{pmatrix}_{n+1} = \begin{pmatrix} \hat{a} \\ \hat{e} \\ \Delta\hat{E}_0 \\ \Delta\hat{E}_1 \end{pmatrix}_n + L_\tau^T \begin{pmatrix} \tilde{r}_0 \\ \tilde{r}_1 \\ \Delta\tilde{\vartheta} \\ \Delta\tilde{t} \end{pmatrix}.$$

It should be noted that setting λ_{ij} $(i=0,1;\ j=\overline{1,4})$ within a unit circle regulates the speed of convergence of the iterative process, the minimum of which is ensured at $\lambda_{ij}=0$.

Numerical calculations. The effectiveness of the proposed method for determining the parameters of elliptical orbits (semi-major axis, eccentricity, eccentric anomalies) by information from on-board measuring instruments, which includes data on the radius vector module of the SC position at the starting point r_0, modulus of the radius-vector position of the SC at the next measuring point r_1, time between measuring points Δt, position angle $\Delta\vartheta$ between radius-vectors r_0 and r_1 was evaluated by the test results shown in Table 1.

Table 1

Test results

r_0, km	r_1, km	Δt, sec	$\Delta\vartheta$, grad	a, km	e	E_0, grad	E_1, grad
6921	7471	190	15	$7.9100 \cdot 10^4$	0.9169	5.6298	9.0375
7471	6921	190	15	$7.9100 \cdot 10^4$	0.9169	351.0538	354.4615
6921	6971	480	30	$6.9434 \cdot 10^3$	0.0139	76.6436	106.6579
6971	6921	480	30	$6.9434 \cdot 10^3$	0.0139	253.4333	283.4477
6919.8	6923.2	479	30.1	$6.9192 \cdot 10^3$	0.0010	95.2723	125.3827
6923.2	6919.8	479	30.1	$6.9192 \cdot 10^3$	0.0010	234.7085	264.8189
6922.8	6922.9	479.2	30.11	$6.9231 \cdot 10^3$	0.00005017	18.6367	48.7454
6921	6971	263	17	$7.4503 \cdot 10^3$	0.0725	11.6389	27.5225
6921	6971	93	6.5	$9.3098 \cdot 10^3$	0.2614	11.0577	16.0809
6921	7371	385	30	$2.1747 \cdot 10^4$	0.6818	0.8378	14.1841
6921	7371	504	36	$1.1774 \cdot 10^4$	0.4122	1.1590	24.9074
7371	6921	504	36	$1.1774 \cdot 10^4$	0.4122	335.1839	358.9323

To ensure the minimum number of search iterations, the components of matrices Φ_1, Φ_0: $\lambda_{11}=\lambda_{12}=\lambda_{13}=\lambda_{14}=\lambda_{01}=\lambda_{02}=\lambda_{03}=\lambda_{04}=0$, were taken equal to zero during the tests. In addition, in the measurements used as input data for the calculation, the time Δt was limited to a value that provides a class of trajectories (Figure *a*) called elliptical trajectories of the first kind [16]. The permissible value Δt was calculated according to the method outlined in [11]. During the algorithm's operation, the observance of the rank condition (7) was monitored at each calculation cycle. If it was not performed, then the parameters of the measurement information were changed. Using the developed algorithm for eight iterations with an accuracy of 10^{-12}, the required parameters given in Table 2 are found. Accordingly, the values a, e, E_0, E_1 by iterations for the first row of Table 1 are given in Table 2.

Table 2

Parameter values by search iterations

Parameter	Iteration								
	0	1	2	3	4	5	6	7	8
A, km	45000	63866	60507	75006	76781	79026	79102	79102	79102
e	0.8544	0.9150	0.8927	0.9171	0.9144	0.9169	0.9169	0.9169	0.9169
E_0, grad	0.1391	0.1007	0.1141	0.0985	0.0999	0.0982	0.0982	0.0982	0.0982
E_1, grad	0.2193	0.1627	0.1827	0.1583	0.1603	0.1577	0.1577	0.1577	0.1577

If we compare the algorithm with the algorithm presented in [2, 8], we can note the following by the results of the simulation proposed here. The developed algorithm has significant advantages in calculation accuracy. Thus, for the initial data (the first line of Table 2) we have $a = 100800$ km, $e = 0.9343$, $E_0 = 4.918°$, $E_1 = 7.933°$, which is significantly worse than the exact values obtained by the developed algorithm. A similar picture for other data (the third line of Table 2 — the calculated data for the algorithm from [2–8] is $a = 7965.5$ km, $e = 0.1602$, $E_0 = 9.17°$, $E_1 = 62.82°$). As for the solution of the Lambert's problem using other algorithms, they have computational advantages in comparison with the developed algorithm. It should be noted that there are other problems of identification (for example, when controlling the approach of SC in the orbit plane), based on the use of systems of algebraic equations of the fourth order, which are solved on board the SC. In this case, on the basis of (9)–(13) it is possible to build a single computing module and thus unify the on-board software. The latter circumstance, as well as the presence of an additional rank control in the algorithm defined by (7), which increases the reliability of the algorithm and determines the preference of the proposed method for solving the Lambert's problem on board a SC within a single turn with respect to other methods.

Conclusion. To solve the Lambert's problem on board the SC developed an algorithm that allows you to apply methods of modal control. It is shown that on the basis of four transcendental algebraic equations (1) it is necessary to construct a discrete model of orbit parameter estimation and using the decomposition method of modal synthesis [12–15] to solve the classical problem of state observer synthesis.

If we look a little wider, then we can assume that in the work proposed approach to the solution of nonlinear algebraic systems of the fourth order, which can be extended to similar (1) systems of any observable order. The peculiarity of the proposed algorithm consists in the fact that the convergence of the iterative process of solution search can have different adjustable speed. Thus, using the considered decomposition method of modal synthesis it is possible to solve not only matrix equations as in [17, 18] but also non-linear algebraic systems of equations.

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